CHAPTER 1 SQUARES AND SQUARE ROOTS

INTRODUCTION

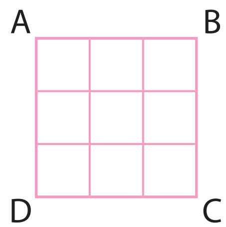
In previous classes, we have already studied the squares of many natural numbers.

For example, $3^2 = 3 \times 3$

We say that 3 to the power 2 or 3 squared is 9.

$$3^2 = 3 \times 3 = 9$$

Now, let us take a square figure ABCD in order to explain the given example. Here, each side of the square has 3 units.



$$\therefore \quad \text{Area} = 3 \times 3 = 3^2 \text{ square units}$$
$$= 9 \text{ square units}$$

In this Chapter, we shall be concentrating on the procedures to find the positive square roots of positive rational numbers.

SQUARES

Look at the examples given below:

$$2 \times 2 = 4 = 2^2$$
$$3 \times 3 = 9 = 3^2$$
$$4 \times 4 = 16 = 4^2$$

Similarly, $a \times a = a^2$

So, we conclude that—

The square of a number is the product obtained by multiplying the number by itself.

Numbers, such as 1, 4, 9, 16, 25, 36 are called **perfect squares**.

Remember

A given number is called a **perfect square** or a **square number** if it is the square of some natural number. These numbers are exact squares and do not involve any decimals or fractions.

To find out whether a given number is a perfect square or not, write the number as a product of its prime factors. If these factors exist in pairs, the number is a perfect square.

Let us take an example to find whether the given number is a perfect square or not.

Example 1: Which of the following numbers are perfect squares?

(i) 256 (ii) 154 (iii) 720

Solution: (i) **Step 1:** $256 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}$

Step 2: Prime factors of 256 can be grouped into pairs and no factor is left out.

$\Rightarrow 256 = (2 \times 2 \times 2 \times 2)^2 = (16)^2$

$\therefore$ 256 is a perfect square of 16.

(ii) **Step 1:** $154 = 2 \times 7 \times 11$

Step 2: No prime factor exists in pairs.

$\therefore$ 154 is not a perfect square.

(iii) **Step 1:** $720 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times 5$

Step 2: In prime factors of 720, factor 5 is left ungrouped.

$\therefore$ 720 is not a perfect square.

Facts About Perfect Squares

(i) A number ending with an odd number of zeroes (one zero, three zeroes and so on) is never a perfect square,

e.g. 150, 25000, 350 are not perfect squares.

(ii) Squares of even numbers are always even,

e.g. $8^2 = 64$ $12^2 = 144$ $20^2 = 400$

(iii) Squares of odd numbers are always odd,

e.g. $7^2 = 49$ $13^2 = 169$ $21^2 = 441$

(iv) The numbers ending with 2, 3, 7, 8 are not perfect squares,

e.g. 32, 243, 37, 368 are not perfect squares.

(v) The square of a number other than 0 and 1, is either a multiple of 3 or exceeds the multiple of 3 by 1.

- Examples of multiples of 3.

  $3^2 = 9$ $\qquad\qquad$ $12^2 = 144$

- Examples of multiples of 3 exceeded by 1.

  $4^2 = 16 = (15 + 1)$ $\qquad\qquad$ $13^2 = 169 = (168 + 1)$

(vi) The square of a number other than 0 and 1, is either a multiple of 4 or exceeds a multiple of 4 by 1.

- Examples of multiples of 4.

  $6^2 = 36$ $\qquad\qquad$ $8^2 = 64$

- Examples of multiples of 4 exceeded by 1.

  $7^2 = 49 = (48 + 1)$ $\qquad\qquad$ $9^2 = 81 = (80 + 1)$

(vii) The difference between the squares of two consecutive natural numbers is equal to their sum.

Let us take two consecutive natural numbers, 3 and 4.

$4^2 - 3^2 = 16 - 9 = 7 = 4 + 3$

Thus, in general, if n and $(n + 1)$ be two consecutive natural numbers,

$$\text{then } (n+1)^2 - n^2 = \left[(n+1)(n+1)\right] - n^2$$
$$= n^2 + n + n + 1 - n^2 = n + (n+1)$$

(viii) The square of a natural number n is equal to the sum of the first n odd natural numbers,

e.g. $1^2 = 1$ = sum of the first one odd natural number

$2^2 = 1 + 3$ = sum of the first two odd natural numbers

$3^2 = 1 + 3 + 5$ = sum of the first three odd natural numbers

$4^2 = 1 + 3 + 5 + 7$ = sum of the first four odd natural numbers and so on.

(ix) Squares of natural numbers composed of only digit 1, follow a peculiar pattern.

$1^2 = 1$

$11^2 = 121$

$111^2 = 12321$

$1111^2 = 1234321$

$11111^2 = 123454321$

We can also observe that the sum of the digits of every such number is a perfect square 1, 121, 12321, 1234321.

$$1 = 1 = 1^2$$
$$1 + 2 + 1 = 4 = 2^2$$
$$1 + 2 + 3 + 2 + 1 = 9 = 3^2$$
$$1 + 2 + 3 + 4 + 3 + 2 + 1 = 16 = 4^2$$
$$1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25 = 5^2$$

See, how beautiful patterns of numbers are made above.

Some Interesting Patterns

Adding triangular numbers

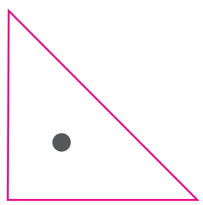
Remember

Numbers whose dot patterns can be arranged as triangles are called **triangular numbers**.

Let us add triangular numbers.

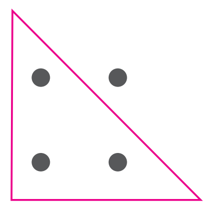
Triangular Numbers

Combining two consecutive triangular numbers



1

$1 + 3 = 4 = 2^2$



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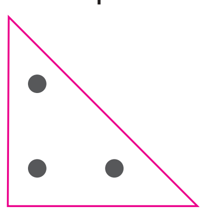

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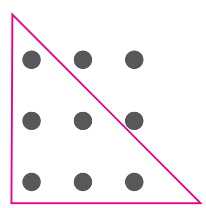
$3 + 6 = 9 = 3^2$



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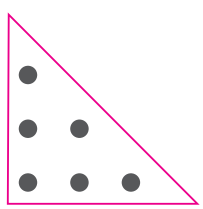

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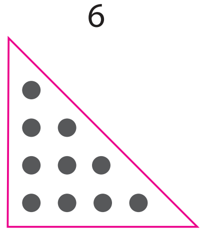
$6 + 10 = 16 = 4^2$



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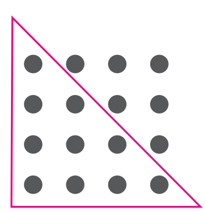

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Observe the following pattern and fill in the blanks.

$$1 + 3 = 2^2$$
$$1 + 3 + 5 = 3^2$$
$$1 + 3 + 5 + 7 = \_\_\_\_\_\_$$
$$1 + 3 + 5 + 7 + 9 = \_\_\_\_\_\_$$
$$1 + 3 + 5 + 7 + 9 + 11 = \_\_\_\_\_\_$$
$$1 + 3 + 5 + 7 + 9 + 11 + 13 = \_\_\_\_\_\_$$

Numbers between square numbers

Let us observe some interesting patterns between two consecutive square numbers.

We have $1^2 = 1$

$2^2 = 4$

The non-square numbers between 1 and 4 are 2, 3.

1, $\boxed{2, 3,}$ 4 $\longrightarrow$ **2 non-square numbers**

The non-square numbers between 4 (= 2^2) and 9 (=3^2) are 5, 6, 7, 8.

4, $\boxed{5, 6, 7, 8,}$ 9 $\longrightarrow$ **4 non-square numbers**

Now, let us put our observations in a tabular form.

Consecutive square numbers	Non-square numbers	Number of non-square numbers
1 and 4	2, 3	2
4 and 9	5, 6, 7, 8	4
9 and 16	10, 11, 12, 13, 14, 15	6
16 and 25	17, 18, ... 24	8
25 and 36	26, 27, ... 35	10

and so on.

Now, let us generalise our observations.

Between 1^2 (= 1) and 2^2 (= 4), there are two non-square numbers (i.e. 2 x 1).

Between 2^2 (= 4) and 3^2 (= 9), there are four non-square numbers (i.e. 2 x 2).

Between 3^2 (= 9) and 4^2 (= 16), there are six non-square numbers (i.e. 2 x 3).

Between 4^2 (= 16) and 5^2 (= 25), there are eight non-square numbers (i.e. 2 x 4).

Between 5^2 (= 25) and 6^2 (= 36), there are ten non-square numbers (i.e. 2 x 5).

Can you say how many non-square numbers are there between 6^2 and 7^2?

We find that if we take any natural number, n and $(n + 1)$, the number of non-square numbers between n^2 and $(n + 1)^2$ is $2n$.

There are $2n$ non-perfect square numbers between the square of the numbers, n and $(n+1)$.

Worksheet 1

1. **Which of the following numbers are perfect squares?**

   11, 16, 32, 36, 50, 64, 75

2. **Which of the following numbers are perfect squares of even numbers?**

   121, 225, 784, 841, 576, 6561

3. **Which of the following numbers are perfect squares?**

   100, 205000, 3610000, 212300000

4. **By just observing the digits at ones place, tell which of the following can be perfect squares?**

   1026, 1022, 1024, 1027

5. **How many non-square numbers lie between the following pairs of numbers?**

   (i) 7^2 and 8^2 (ii) 10^2 and 11^2 (iii) 40^2 and 41^2

   (iv) 80^2 and 81^2 (v) 101^2 and 102^2 (vi) 205^2 and 206^2

6. **Write down the correct number in the box.**

   (i) $100^2 - 99^2 = \boxed{} = \boxed{}$

   (ii) $27^2 - 26^2 = \boxed{} = \boxed{}$

   (iii) $569^2 - 568^2 = \boxed{} = \boxed{}$

7. **Observe the pattern in the following and find the missing numbers.**

$$1\underline{2}1 = \frac{(22)^2}{1+2+1}$$

$$12\underline{3}21 = \frac{(333)^2}{1+2+3+2+1}$$

$$123\underline{4}321 = \_\_\_\_\_\_\_\_$$

$$1234\underline{5}4321 = \_\_\_\_\_\_\_\_$$

$$12345\underline{6}54321 = \_\_\_\_\_\_\_\_$$

8. **Which of the following triplets are Pythagorean?**

(3, 4, 5), (6, 7, 8), (10, 24, 26), (2, 3, 4)

[**Hint** : Let the smallest even number be $2m$ and find m from it. Then, find ($2m$, $m^2 - 1$, $m^2 + 1$). If you get the triplet, it is Pythagorean.]

Another way of finding a Pythagorean triplet is:

If '*a*', '*b*' and '*c*' are three natural numbers with '*a*' as the smallest of them, then,

(i) If '*a*' is odd, sum of other two numbers is a^2 and their difference is 1.

(ii) If '*a*' is even, sum of other two numbers is $\frac{a^2}{2}$ and their difference is 2.

SQUARE ROOTS

We know that

$$4^2 = 4 \times 4 = 16$$

We say square root of 16 is 4.

This is written as $\sqrt{16} = 4$.

Note: $(-4)^2 = 16$ Therefore, square root of 16 can be −4 also, but here we are taking only positive square root.

Let us see some more examples.

$$7^2 = 49 \longrightarrow \sqrt{49} = 7$$

$$5^2 = 25 \longrightarrow \sqrt{25} = 5$$

$$8^2 = 64 \longrightarrow \sqrt{64} = 8$$

In general, if $m^2 = n$ then $\sqrt{n} = m$

Hence, square root of a given natural number n is that natural number m whose square is n.

From the above examples, we observe that—

(i) the square root of an even number is even.

(ii) the square root of an odd number is odd.

(iii) the symbol for the square root is $\sqrt{\ }$.

(iv) if a is the square root of b, then b is the square of a.

Observe the following pattern.

$$1 + 3 = 2^2$$

$$1 + 3 + 5 = 3^2$$

$$1 + 3 + 5 + 7 = 4^2$$

$$1 + 3 + 5 + 7 + 9 + 11 + 13 = 7^2$$

$$1 + 3 + 5 + 7 + \ldots\ldots\ldots\ldots \text{ up to } n \text{ terms} = n^2$$

The sum of first n odd numbers is n^2.

Finding Square Root of a Number by the Repeated Subtraction Method

Let us find $\sqrt{9}$.

Step 1: $9 - 1 = 8$ → First odd number

Step 2: $8 - 3 = 5$ → Second odd number

Step 3: $5 - 5 = 0$ → Third odd number

We have subtracted from 9, the successive odd numbers 1, 3 and 5, and obtained 0 in **Step 3**.

$\therefore \quad \sqrt{9} = 3$

Consider another example.

Example 2: Find $\sqrt{121}$ by repeated subtraction method.

Solution : **Step 1**: $121 - 1 = 120$ **Step 3**: $117 - 5 = 112$

Step 2: $120 - 3 = 117$ **Step 4**: $112 - 7 = 105$

Step 5: $105 - 9 = 96$

Step 6: $96 - 11 = 85$

Step 7: $85 - 13 = 72$

Step 8: $72 - 15 = 57$

Step 9: $57 - 17 = 40$

Step 10: $40 - 19 = 21$

Step 11: $21 - 21 = 0$

We have subtracted from 121, the successive odd numbers from 1 to 21, and obtained 0 in **Step 11**.

$\therefore \quad \sqrt{121} = 11$

Worksheet 2

Find the square root of the following numbers by the repeated subtraction method.

(i)	16	(ii)	49	(iii)	64	(iv)	100
(v)	169	(vi)	81	(vii)	256	(viii)	144

Finding Square Root by Prime Factorisation

To find the square root of a perfect square by prime factorisation, we go through the following steps:

I. Write down the prime factors of the given number.

II. Make pairs of prime factors such that both the factors in each pair are equal.

III. Write one factor from each pair.

IV. Find the product of the above factors.

V. The product is the required square root.

Let us now take some examples to find the square root by prime factorisation.

Example 3: Find the square root of 1156.

Solution:

$$1156 = \underline{2 \times 2} \times \underline{17 \times 17}$$

$$\sqrt{1156} = \sqrt{\underline{2\times2}\times\underline{17\times17}}$$

$$\sqrt{1156} = 2 \times 17$$

$$= 34$$

2	1156
2	578
17	289
17	17
	1

Therefore, the square root of 1156 is 34.

Example 4: Find the square root of 11025.

3	11025
3	3675
5	1225
5	245
7	49
7	7
	1

Solution:

$$11025 = \underline{3 \times 3} \times \underline{5 \times 5} \times \underline{7 \times 7}$$

$$\sqrt{11025} = \sqrt{\underline{3 \times 3} \times \underline{5 \times 5} \times \underline{7 \times 7}}$$

$$\sqrt{11025} = 3 \times 5 \times 7$$

$$= 105$$

Therefore, the square root of 11025 is 105.

Example 5: Find the smallest number by which 9408 must be divided so that it becomes a perfect square. Also, find the square root of the number so obtained.

2	9408
2	4704
2	2352
2	1176
2	588
2	294
7	147
7	21
3	3
	1

Solution:

$$9408 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7} \times 3$$

We observe that prime factor 3 does not form a pair.

Therefore, we must divide the number by 3 so that the quotient becomes a perfect square.

$$\therefore \quad \frac{9408}{3} = 3136$$

$$3136 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (7 \times 7)$$

Now, each prime factor occurs in pairs. Therefore, the required smallest number is 3.

$$\therefore \quad \sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$$

Worksheet 3

1. **Find the square root of each of the following by prime factorisation.**

   (i) 225 (ii) 441 (iii) 529 (iv) 40000

   (v) 7744 (vi) 8281 (vii) 4096 (viii) 28900

2. **Find the smallest number by which 1100 must be multiplied so that the product becomes a perfect square. Also, find the square root of the perfect square so obtained.**

3. **By what smallest number must 180 be multiplied so that it becomes a perfect square? Also, find the square root of the number so obtained.**

4. **Find the smallest number by which 3645 must be divided so that it becomes a perfect square. Also, find the square root of the resulting number.**

5. **A gardener planted 1,521 trees in rows such that the number of rows was equal to the number of plants in each row. Find the number of rows.**

6. **An officer wants to arrange 2,02,500 cadets in the form of a square. How many cadets were there in each row?**

7. **The area of a square field is 5184 m^2. A rectangular field, whose length is twice its breadth, has its perimeter equal to the perimeter of the square field. Find the area of the rectangular field.**

8. **Find the value of $\sqrt{47089} + \sqrt{24336}$**

Finding Square Root by Long Division Method

When numbers are very large, the method of finding their square roots by prime factorisation becomes lengthy. So, we use long division method. Consider the following steps to find the square root of any number, say 1521.

Step 1: Mark off the digits in pairs starting with the ones digit. Each pair and remaining one digit (if it is there) is called a **period**.

$$\begin{array}{|l} \overline{15}\,\overline{21} \end{array}$$

Step 2: Think of the largest number whose square is either equal to or just less than the first period starting from the left. This digit is the quotient as well as the divisor. Put the quotient above the period and write the product of divisor and quotient just below the first period.

$$\begin{array}{r|l} & 3 \\ \hline 3 & \overline{15}\,\overline{21} \\ & 9 \\ \hline \end{array}$$

Step 3: Find the remainder (6 in this case).

$$\begin{array}{r|l} & 3 \\ \hline 3 & \overline{15}\,\overline{21} \\ & 9 \\ \hline & 6 \end{array}$$

Step 4: Bring down the next pair of digits (i.e. second period) to the right of the remainder. This becomes the new dividend (i.e. 621).

$$\begin{array}{r|l} & 3 \\ \hline 3 & \overline{15}\,\overline{21} \\ & 9\downarrow \\ \hline & 6\,21 \end{array}$$

Step 5: Double the current quotient and enter it as divisor with a blank on its right.

$$\begin{array}{r|l} & 3 \\ \hline 3 & \overline{15}\,\overline{21} \\ & 9 \\ \hline (3 \times 2) \leftarrow 6\_ & 6\;21 \end{array}$$

Step 6: Guess a largest possible digit to fill the blank which also becomes the new digit in the quotient such that when the new digit is multiplied to the new divisor, the product is either less than or equal to the dividend.

$$\begin{array}{r|l}
 & \ \ 3\ \ 9 \\ \hline
3 & \overline{15}\ \overline{21} \\
 & \ \ 9 \downarrow \\ \hline
6\underline{9} & \ \ 6\ 21 \\
 & \ \ 6\ 21 \longrightarrow (69 \times 9) \\ \hline
 & \quad 0
\end{array}$$

Step 7: Now, subtract the product of new divisor and the new digit from the new dividend.

Step 8: If the remainder is zero and no period is left, then we stop and the current quotient is the square root of the given number (like in this case). So here, $\sqrt{1521} = 39$.

And if the remainder is non-zero, then repeat the Steps from 5 to 8 till all the periods have been taken care of.

Let us look at another example.

Example 6: Find $\sqrt{99856}$

Solution:

Step 1:

$$\begin{array}{|l}
\overline{9}\ \overline{98}\ \overline{56}
\end{array}$$

Step 2:

$$\begin{array}{r|l}
 & 3 \\ \hline
3 & \overline{9}\ \overline{98}\ \overline{56} \\
 & -9 \downarrow \\ \hline
 & 0\ 98
\end{array}$$

Step 3:

$$\begin{array}{r|l}
 & 3_ \\ \hline
3 & \overline{9}\ \overline{98}\ \overline{56} \\
 & -9 \downarrow \\ \hline
(3 \times 2) \longleftarrow 6_ & 0\ 98
\end{array}$$

Step 4:

$$\begin{array}{r|l}
 & 3\ \underline{1} \\ \hline
3 & \overline{9}\ \overline{98}\ \overline{56} \\
 & -9 \downarrow \\ \hline
6\underline{1} & 0\ 98 \\
 & -\ 61 \longrightarrow (61 \times 1) \\ \hline
 & \ \ 3756
\end{array}$$

Step 5:

$$\begin{array}{r|l}
 & 3\ 1\underline{6} \\ \hline
3 & \overline{9}\ \overline{98}\ \overline{56} \\
 & -9 \downarrow \\ \hline
61 & 0\ 98 \\
 & -\ 61 \\ \hline
62\underline{6} & \ \ 3756 \\
(31 \times 2) \longleftarrow & -\ \underline{3756} \longrightarrow (626 \times 6) \\
 & \quad 0
\end{array}$$

$\therefore \quad \sqrt{99856} = 316$

Now, let us try to understand long division method of square roots by some more examples.

Example 7: Find the square root of 4401604.

Solution:

$$
\begin{array}{r|l}
 & \quad 2098 \\
\hline
2 & 4\ \overline{40}\ \overline{16}\ \overline{04} \\
 & -4 \\
\hline
40 & \quad 40 \\
 & -\ \ 00 \\
\hline
\underbrace{409}_{(20 \times 2)} & \quad 4016 \\
 & -\ \ 3681 \longrightarrow (409 \times 9) \\
\hline
\underbrace{4188}_{(209 \times 2)} & \quad 33504 \\
 & -\ \ 33504 \longrightarrow (4188 \times 8) \\
\hline
 & \qquad 0
\end{array}
$$

$\therefore \quad \sqrt{4401604} = 2098$

Example 8: Find the square root of 1734489 by long division method.

Solution: Apply long division method to find square root of 1734489.

$$
\begin{array}{r|l}
 & \quad 1317 \\
\hline
1 & 1\ \overline{73}\ \overline{44}\ \overline{89} \\
 & -1 \\
\hline
23 & 0\ 73 \\
 & -\ \ 69 \\
\hline
261 & \quad 444 \\
 & -\ \ 261 \\
\hline
2627 & \quad 18389 \\
 & -\ \ 18389 \\
\hline
 & \qquad 0
\end{array}
$$

$\therefore \quad \sqrt{1734489} = 1317$

Example 9: Find the least number which must be subtracted from 7581 to obtain a perfect square. Find the perfect square and its square root.

Solution:

$$
\begin{array}{r|l}
 & \quad 87 \\
\hline
8 & \overline{75}\ \overline{81} \\
 & -64 \\
\hline
167 & \quad 1181 \\
 & -1169 \\
\hline
 & \qquad 12
\end{array}
$$

$\therefore$ 12 should be subtracted from 7581 to make it a perfect square.

Hence, the perfect square = 7581 – 12 = 7569

and $\sqrt{7569} = 87$

Example 10: What least number must be added to 5607 to make the sum a perfect square? Find the perfect square and its square root.

Solution: Try to find the square root of 5607.

$$\begin{array}{r|r} & 74 \\ \hline 7 & \overline{56}\ \overline{07} \\ & -49 \\ \hline 144 & 707 \\ & -576 \\ \hline & 131 \end{array} \qquad \begin{array}{r|r} & 75 \\ \hline 7 & \overline{56}\ \overline{07} \\ & -49 \\ \hline 145 & 707 \\ & -725 \\ \hline & -18 \end{array}$$

We observe that $(74)^2 < 5607 < (75)^2$

$\therefore$ 5607 is (725 – 707) = 18 less than $(75)^2$.

So, we must add 18 to 5607 to make it a perfect square.

Hence, the perfect square = 5607 + 18 = 5625 and $\sqrt{5625} = 75$

Worksheet 4

1. **Find the square root of the following numbers by the long division method.**

(i) 9801	(ii) 6561	(iii) 390625	(iv) 108241
(v) 363609	(vi) 120409	(vii) 1471369	(viii) 57121

2. **Find the least number which must be subtracted from 6203 to obtain a perfect square. Also, find square root of the number so obtained.**
3. **Find the greatest number of six digits which is a perfect square. Find the square root of this number.**
4. **Find the least number which must be added to 6203 to obtain a perfect square. Also, find the square root of the number so obtained.**
5. **Find the least number of six digits which is a perfect square. Find the square root of this number.**
6. **Find the value of $\sqrt{64432729} - \sqrt{9653449}$**

SQUARE ROOT OF A RATIONAL NUMBER

In this Chapter, we shall be taking some examples to understand the rules of finding the square root of rational numbers.

Example 11: Find $\sqrt{49 \times 25}$ and show that it is equal to $\sqrt{49} \times \sqrt{25}$.

Solution:

$$\sqrt{49 \times 25} = \sqrt{7^2 \times 5^2}$$

$$= \sqrt{7^2 \times 5^2} \qquad \text{[We know } a^m \times b^m = (ab)^m\text{]}$$

$$= \sqrt{(35)^2} = 35 = 7 \times 5$$

$$= \sqrt{49} \times \sqrt{25}$$

Rule I: For perfect squares a and b, $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$.

Example 12: Consider $\sqrt{\frac{49}{25}}$ and $\frac{\sqrt{49}}{\sqrt{25}}$ and find out whether they are equal.

Solution:

$$\sqrt{\frac{49}{25}} = \sqrt{\frac{7^2}{5^2}}$$

$$\left\{\text{We know} \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m},\ b \neq 0\right\} = \sqrt{\left(\frac{7}{5}\right)^2} = \frac{7}{5}$$

Also, $\frac{\sqrt{49}}{\sqrt{25}} = \frac{\sqrt{7^2}}{\sqrt{5^2}}$

$$= \frac{7}{5}$$

Thus, $\sqrt{\frac{49}{25}} = \frac{7}{5} = \frac{\sqrt{49}}{\sqrt{25}}$

Rule II: For perfect squares a and b, where $b \neq 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Now, let us apply Rule I and II and solve some examples.

Example 13: Find the value of $\frac{\sqrt{243}}{\sqrt{867}}$

Solution:

$$\frac{\sqrt{243}}{\sqrt{867}} = \sqrt{\frac{243}{867}} \qquad \text{(using Rule II)}$$

$$= \sqrt{\frac{81}{289}} \qquad \text{(cancel the common factor 3)}$$

$$= \frac{\sqrt{81}}{\sqrt{289}}$$

$$= \frac{\sqrt{9^2}}{\sqrt{17^2}}$$

$$= \frac{9}{17}$$

Thus, the value of $\frac{\sqrt{243}}{\sqrt{867}} = \frac{9}{17}$.

Example 14: Find the square root of:

(i) $1\frac{56}{169}$ (ii) 14400

Solution: (i) $\sqrt{1\frac{56}{169}} = \sqrt{\frac{225}{169}}$

$$= \frac{\sqrt{225}}{\sqrt{169}} = \frac{\sqrt{15^2}}{\sqrt{13^2}} \qquad \text{(by Rule II)}$$

$$= \frac{15}{13}$$

$$= 1\frac{2}{13}$$

(ii) $\sqrt{14400} = \sqrt{144 \times 100}$

$$= \sqrt{144} \times \sqrt{100} \qquad \text{(by Rule I)}$$

$$= \sqrt{12^2} \times \sqrt{10^2}$$

$$= 12 \times 10$$

$$= 120$$

SQUARES OF TERMINATING DECIMALS

Observe some squares.

4^2	=	16	and	$(0.4)^2$	=	0.16
12^2	=	144	and	$(1.2)^2$	=	1.44
15^2	=	225	and	$(0.15)^2$	=	0.0225
18^2	=	324	and	$(0.18)^2$	=	0.0324

From the above examples, we observe that the square of a decimal consists of twice as many decimal places as given in the number.

Example 15: Find the square of:

(i) 0.41 (ii) 2.6 (iii) 0.25 (iv) 0.009

Solution:

(i) $(0.41)^2 = 0.1681$

(ii) $(2.6)^2 = 6.76$

(iii) $(0.25)^2 = 0.0625$

(iv) $(0.009)^2 = 0.000081$

Square Roots of Numbers in Decimal Form (which are perfect squares)

Let us find the square root of a decimal number.

Example 16: Find the square root of 147.1369

Solution:

$$
\begin{array}{r|l l}
 & 12.13 & \\
\hline
1 & \overline{1}\;\overline{47}\,.\,\overline{13}\;\overline{69} & \\
 & -1 & \\
\hline
22 & 0\,47 & \\
 & -44 & \longrightarrow (22 \times 2) \\
\hline
241 & 313 & \\
 & -241 & \longrightarrow (241 \times 1) \\
\hline
2423 & 7269 & \\
 & -7269 & \longrightarrow (2423 \times 3) \\
\hline
 & 0 &
\end{array}
$$

Therefore, $\sqrt{147.1369} = 12.13$

From the above example, the steps of finding out square roots of numbers in decimal form are clear.

Step 1: In the whole number part, make pairs from the right. But in the decimal part, make pairs from the left.

Step 2: Then, find square root as in the case of long division method.

Step 3: Place the decimal point as soon as the integral part comes to an end.

Observe that above steps are taken in the following example also.

Example 17: Find the square root of 0.00059049

Solution:

$$
\begin{array}{r|r}
 & 0.0243 \\
\hline
0 & 0.\overline{00}\ \overline{05}\ \overline{90}\ \overline{49} \\
 & -00 \\
\hline
2 & 05 \\
 & -\ 4 \\
\hline
44 & 190 \\
 & -176 \\
\hline
483 & 1449 \\
 & -1449 \\
\cline{2-2}
 & 0
\end{array}
$$

Therefore, $\sqrt{0.00059049} = 0.0243$

Approximate Value of the Square Roots of Natural Numbers (which are not perfect squares)

Long division method is also used to find approximate square roots of numbers or decimals up to certain decimal places. Let us look at the following examples.

Example 18: Find the square root of 3 up to three decimal places.

Solution: To find the number up to three decimal places which is equal to $\sqrt{3}$, we add three pairs of zeroes (six zeroes) to the right of decimal point.

$$
\begin{array}{r|r}
 & 1.732 \\
\hline
1 & 3.\ \overline{00}\ \overline{00}\ \overline{00} \\
 & -1 \\
\hline
27 & 200 \\
 & -189 \\
\hline
343 & 1100 \\
 & -1029 \\
\hline
3462 & 7100 \\
 & -6924 \\
\cline{2-2}
 & 176
\end{array}
$$

Hence, $\sqrt{3} = 1.732$ up to three decimal places.

Example 19: Find the square root of $2\frac{1}{5}$ correct to two places of decimal.

Solution: $2\frac{1}{5} = \frac{11}{5} = 2.2$

$$\sqrt{2\frac{1}{5}} = \sqrt{2.2}$$

$$\begin{array}{r|r}
 & 1.483 \\ \hline
1 & 2.\overline{20}\;\overline{00}\;\overline{00} \\
 & -1 \\ \hline
24 & 120 \\
 & -96 \\ \hline
288 & 2400 \\
 & -2304 \\ \hline
2963 & 9600 \\
 & -8889 \\ \hline
 & 711
\end{array}$$

$$\therefore \sqrt{2\frac{1}{5}} = \sqrt{2.2} = 1.483$$

$$\cong 1.48 \text{ (correct to two places of decimal)}$$

> **Note:** We were required to find the square root of $2\frac{1}{5}$ correct to two places of decimal. Here, we have found the square root up to three places of decimal. In the third place, we have 3 (<5) and therefore, in the final result, 3 is ignored.

Square Root of Other Numbers (not perfect squares) by Estimation

It is easy to work out the square root of a perfect square, but it is really hard to work out the square root of other numbers. Well, in such cases, we need to **estimate** the square root. Let us do some examples.

Example 20: Find the square root of 10 by estimation.

Solution: The perfect squares near to 10 are 9 and 16,

i.e. $9 < 10 < 16$ or

$$3^2 < 10 < 4^2$$

So, we can guess that the answer is between 3 and 4, i.e. $3 < \sqrt{10} < 4$

Let us try with 3.5 as $3 < 3.5 < 4$

But $3.5 \times 3.5 = 12.25 > 10$

i.e. $3^2 < 10 < (3.5)^2$

Let us further reduce the number 3.5 to 3.2

So, $3.2 \times 3.2 = 10.24 > 10$

i.e. $3^2 < 10 < (3.2)^2$

Let us try with 3.1 so that

$$3.1 \times 3.1 = 9.61$$

So, we can say $9.61 < 10 < 10.24$

or $(3.1)^2 < 10 < (3.2)^2$

But 10.24 is much closer to 10 as compared to 9.61.

So, we can say $\sqrt{10}$ is 3.2 approximately.

Example 21: Find the square root of 410 by estimation.

Solution: The perfect square near to 410 are 400 and 441

i.e. $400 < 410 < 441$

$$20^2 < 410 < 21^2$$

We guess the answer is between 20 and 21

Let us try with 20.3 as $20 < 20.3 < 21$

But $(20.3)^2 = 412.09 > 410$

i.e. $20^2 < 410 < (20.3)^2$

Let us try 20.2

$$20.2 \times 20.2 = 408.04$$

$$408.04 < 410 < 412.09$$

$$(20.2)^2 < 410 < (20.3)^2$$

We take $\sqrt{410}$ as 20.2 approximately.

Worksheet 5

1. **Find the square root of the following fractions.**

   (i) $\frac{324}{361}$ (ii) $\frac{441}{961}$ (iii) $5\frac{19}{25}$ (iv) $21\frac{51}{169}$

   (v) $\frac{5625}{441}$ (vi) $7\frac{18}{49}$ (vii) $23\frac{394}{729}$ (viii) $35\frac{85}{1444}$

2. **Find the value of:**

   (i) $\sqrt{0.0009}$ (ii) $\sqrt{0.0081}$ (iii) $\sqrt{0.012321}$ (iv) $\sqrt{7.29}$

3. **Find the square root of:**

   (i) 0.053361 (ii) 0.00053361 (iii) 150.0625

   (iv) 0.374544 (v) 610.09

4. **Find the square root of the following (correct to three decimal places).**

   (i) 7 (ii) 2.5 (iii) $2\frac{1}{12}$ (iv) $367\frac{2}{7}$

5. **Estimate the value of the following to the nearest to one decimal place.**

   (i) $\sqrt{90}$ (ii) $\sqrt{150}$ (iii) $\sqrt{600}$ (iv) $\sqrt{1000}$

6. **Devika has a square piece of cloth of area 9 m^2 and she wants to make 16 square-shaped scarves of equal size out of it. What should possibly be the length of the side of the scarf that can be made out of this piece?**

7. **The area of a square plot is 800 m^2. Find the estimated length of the side of the plot.**

Value Based Questions

1. **Priya wants to wish her teacher on Teacher's Day by giving her a self-made greeting card. She chooses a pink coloured square sheet of paper. A side of that paper measures 19.5 cm.**

   (a) Find the area of paper she chooses for the card.

   (b) What act of Priya did you like?

2. The students of Class-VIII B of a school donated ₹ 2,304 for the Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class.

(a) Find the number of students in VIII B.

(b) What quality of the students do you appreciate here?

Brain Teasers

1.A. Tick (✓) the correct option.

(a) The difference between the squares of two consecutive number is equal to their—

(i) difference (ii) sum (iii) product (iv) quotient

(b) What will be the digit in the thousands place of $(1111)^2$?

(i) 3 (ii) 4 (iii) 2 (iv) 1

(c) Perfect squares cannot have 2, 3, __ and __ in its ones place.

(i) 1, 7 (ii) 5, 6 (iii) 7, 8 (iv) 7, 9

(d) The smallest number by which 72 must be divided to make it a perfect square is—

(i) 4 (ii) 5 (iii) 3 (iv) 2

(e) The square root of 3.052009 has _____ decimal places.

(i) 3 (ii) 4 (iii) 5 (iv) 1

B. Answer the following questions.

(a) How many non–square numbers are there between 13^2 and 14^2?

(b) Write the first four triangular numbers.

(c) Is 5, 7, 9 a Pythagorean triplets? Why? Justify.

(d) Find $\sqrt{9}$ by repeated subtraction method.

(e) Find the measure of the side of a square handkerchief of area 324 cm^2.

2. **Find the square root of 10 correct to four places of decimal.**

3. **Find the values of : $\sqrt{3.1428}$ and $\sqrt{0.31428}$ correct to three decimal places.**

4. **Simplify:**

   (i) $\dfrac{\sqrt{0.0441}}{\sqrt{0.000441}}$ (ii) $\sqrt{49}+\sqrt{0.49}+\sqrt{0.0049}$

5. **The area of a square field is $101\dfrac{1}{400}$ m^2. Find the length of one side of the field.**

6. **What is that number which when multiplied by itself gives 227.798649?**

7. **In a lecture hall, 8,649 students are sitting in such a manner that there are as many students in a row as there are rows in the lecture hall. How many students are there in each row of the lecture hall?**

8. **A General wishing to draw up his 64,019 men in the form of a square found that he had 10 men extra. Find the number of men in the front row.**

HOTS

1. **The cost of levelling a square lawn at ₹ 15 per square metre is ₹ 19,935. Find the cost of fencing the lawn at ₹ 22 per metre.**

2. **If $\sqrt{2}=1.414$, $\sqrt{5}=2.236$ and $\sqrt{3}=1.732$, find the value of:**

   (i) $\sqrt{72}+\sqrt{48}$ (ii) $\sqrt{\dfrac{125}{64}}$

Enrichment Questions

1. **The product of two numbers is 1296. If one number is 16 times the other, find the number.**

2. **Find the value of $\sqrt{50625}$ and hence the value of $\sqrt{506.25}+\sqrt{5.0625}$.**

3. **Write a Pythagorean triplet if one number is 14.**

1. The square of a number is that number raised to the power 2.

2. A square number is never negative.

3. A number ending in 2, 3, 7 or 8 is never a perfect square.

4. (i) Squares of even numbers are even.

   (ii) Squares of odd numbers are odd.

5. A perfect square number leaves a remainder 0 or 1 on division by 3.

6. There are no natural numbers p and q such that $p^2 = 2q^2$.

7. If a and b are perfect squares ($b \neq 0$), then

   $$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

   and $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$.

8. The square root of a perfect square can be obtained by —

   (i) finding prime factors.

   (ii) long division method.

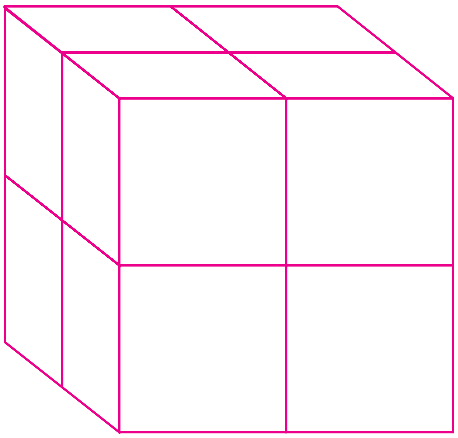
9. The pairing of numbers in the division method starts from a decimal point. For the integral part, it goes from right to left and for the decimal part, it goes from left to right.

10. If p and q are not perfect squares, then to find $\sqrt{\dfrac{p}{q}}$, we express $\dfrac{p}{q}$ as a decimal and then apply division method to find the square root.

CHAPTER 2 CUBES AND CUBE ROOTS

INTRODUCTION

In this Chapter, we shall confine ourselves to exponent **three**, that is $n^3 = n \times n \times n$. It is called **cube** of n.



Now, let us take a cube of 2 units and examine it. Each side of a cube is of 2 units.

Therefore, the volume of a cube having 2 units

$$= 2 \times 2 \times 2 = 2^3 \text{ cubic units}$$

$$= 8 \text{ cubic units}$$

We say that 2 to the power of 3 or 2 cubed is 8.

Now, let us study cubes of rational numbers and cube roots of those rational numbers which are perfect cubes. In the previous chapter, we have taken only the square roots of positive rational numbers, whereas in this Chapter, we shall be studying the cube roots of positive as well as negative rational numbers.

CUBES

Let us observe the following products:

$$2 \times 2 \times 2 = 8 \longrightarrow 8 = 2^3$$

$$3 \times 3 \times 3 = 27 \longrightarrow 27 = 3^3$$

Similary, $a \times a \times a = a^3$

So, we conclude that—

The cube of a number is product of a number multiplied by itself three times and is read as the number raised to the power 3.

Observe the cubes of following numbers.

$$4^3 = 4 \times 4 \times 4 = 64$$

$$8^3 = 8 \times 8 \times 8 = 512$$

$$12^3 = 12 \times 12 \times 12 = 1728$$

$$17^3 = 17 \times 17 \times 17 = 4913$$

We see that 64, 512, 1728 and 4913 are cubes of some positive integers. Such numbers are called **perfect cubes**.

An integer *n* is a perfect cube if there is an integer *m* such that $n = m \times m \times m$.

In order to check whether a given number is a perfect cube, we follow the given steps:

Step 1: Express the given number as a product of prime factors.

Step 2: Group the factors in triplets such that all three factors in each triplet are the same.

Step 3: If some prime factors are left ungrouped, the given number is not a perfect cube.

Now, let us examine some numbers for being perfect cubes.

Example 1: Is 216 a perfect cube?

Solution: Resolve 216 into prime factors.

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

Group them in like factors.

$$216 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$$

$$= 2^3 \times 3^3$$

Prime factors of 216 can be grouped into triplets and no factor is left over.

$\therefore$ 216 is a perfect cube.

Example 2: Is 1323 a perfect cube?

Solution: Resolve 1323 into prime factors.

3	1323
3	441
3	147
7	49
7	7
	1

$$1323 = \underline{3 \times 3 \times 3} \times 7 \times 7$$

While grouping the factors, we are left with 7×7.

Therefore, 1323 is not a perfect cube.

Example 3: What is the smallest number by which 3087 may be multiplied so that the product is a perfect cube?

3	3087
3	1029
7	343
7	49
7	7
	1

Solution: Find prime factors of 3087.

$$3087 = 3 \times 3 \times \underline{7 \times 7 \times 7}$$

On grouping the factors, we find that 3×3 is left out.

So, if we multiply 3087 by 3, the factors would be

$\underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$. [3 appears three times]

Hence, 3087 must be multiplied by 3 to make it a perfect cube.

Example 4: What is the smallest number by which 3087 must be divided so that the quotient is a perfect cube?

3	3087
3	1029
7	343
7	49
7	7
	1

Solution: Resolve 3087 into prime factors.

$$3087 = 3 \times 3 \times \underline{7 \times 7 \times 7}$$

On grouping the factors, we find that 3×3 is left out.

So, we divide 3087 by $3 \times 3 = 9$.

The quotient then would be $7 \times 7 \times 7$, which is a perfect cube.

$\therefore$ 3087 must be divided by 9 so that the quotient becomes a perfect cube.

Properties of Cubes of Numbers

Let us now keep in mind, cubes of some numbers, i.e.

$2^3 = 8, 3^3 = 27, 4^3 = 64, 5^3 = 125, 6^3 = 216, ...$

We will observe that the following properties are true:

I. The cube of an even number is always even, whereas the cube of an odd number is always odd.

II. The cube of any multiple of 2 is divisible by 8.

$(4)^3 = 64$ which is divisible by 8.

$(12)^3 = 1728$ which is divisible by 8.

III. The cube of any multiple of 3 is divisible by 27.

$(9)^3 = 729$ which is divisible by 27.

$(12)^3 = 1728$ which is divisible by 27.

IV. The cube of a negative number is always negative.

$(-2)^3 = (-2) \times (-2) \times (-2) = 4 \times (-2) = -8$

$(-5)^3 = (-5) \times (-5) \times (-5) = 25 \times (-5) = -125$

V. The cube of a positive number is always positive.

VI. The cube of a rational number is equal to the cube of its numerator divided by the cube of its denominator.

$$\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3} = \frac{27}{125}$$

Worksheet 1

1. Find the cubes of:

(i) 8 (ii) 13 (iii) 17 (iv) 1.3

(v) 0.06 (vi) 0.4 (vii) $\frac{2}{3}$ (viii) – 7

(ix) – 9 (x) – 12

2. Which of the following numbers are perfect cubes?

(i) 4096 (ii) 108 (iii) 392

(iv) – 27000 (v) $\frac{-64}{1331}$

3. Find the smallest number by which 2560 must be multiplied so that the product is a perfect cube.

4. Find the smallest number by which 8788 be divided so that the quotient is a perfect cube.

5. Write True or False for the following statements.

(i) 650 is not a perfect cube. ________

(ii) Perfect cubes may end with two zeroes. ________

(iii) Perfect cubes of odd numbers may not always be odd. ________

(iv) Cube of negative numbers are negative. ________

(v) For a number to be a perfect cube, it must have prime factors in triplets. ________

CUBE ROOTS

We know that

$$2^3 = 2 \times 2 \times 2 = 8$$

We say cube root of 8 is 2

We write $\sqrt[3]{8} = 2$

Let us see more examples.

$$3^3 = 27 \longrightarrow \sqrt[3]{27} = 3$$

$$4^3 = 64 \longrightarrow \sqrt[3]{64} = 4$$

$$5^3 = 125 \longrightarrow \sqrt[3]{125} = 5$$

$$(-10)^3 = -1000 \longrightarrow \sqrt[3]{-1000} = -10$$

$$(-7)^3 = -343 \longrightarrow \sqrt[3]{-343} = -7$$

In general if, $m^3 = n$ then $\sqrt[3]{n} = m$.

Hence, cube root of a given number n is that number m whose cube is n.

Note: '$\sqrt[3]{\ }$' symbol denotes cube root whereas '$\sqrt{\ }$' denotes square root.

Some Patterns Involving Cubic Numbers

I. Let us look at some interesting patterns expressing cubic numbers.

$1^3 = 1$

$2^3 = 3 + 5$

$3^3 = 7 + 9 + 11$

$4^3 = 13 + 15 + 17 + 19$

$5^3 = 21 + 23 + 25 + 27 + 29$

and so on.

What do we observe from the above pattern?

We see that the cube of a number n can be expressed as the sum of the n odd consecutive numbers.

II. Let us see another pattern.

$$2^3 - 1^3 = 1 + 2 \times 1 \times 3$$
$$3^3 - 2^3 = 1 + 3 \times 2 \times 3$$
$$4^3 - 3^3 = 1 + 4 \times 3 \times 3$$
$$5^3 - 4^3 = 1 + 5 \times 4 \times 3$$
$$6^3 - 5^3 = 1 + 6 \times 5 \times 3$$

and so on.

III. Now, let us look at this pattern.

$$1^3 = 1$$
$$2^3 - 1^3 = 8 - 1 = 7$$
$$3^3 - 2^3 = 27 - 8 = 19$$
$$4^3 - 3^3 = 64 - 27 = 37$$
$$5^3 - 4^3 = 125 - 64 = 61$$
$$6^3 - 5^3 = 216 - 125 = 91$$

$\therefore$
$$1^3 = 1$$
$$2^3 = 1 + 7$$
$$3^3 = 1 + 7 + 19$$
$$4^3 = 1 + 7 + 19 + 37$$
$$5^3 = 1 + 7 + 19 + 37 + 61$$
$$6^3 = 1 + 7 + 19 + 37 + 61 + 91$$

and so on.

The last number in each case, that is 1, 7, 19, 37, 61, ... may be obtained by putting $n = 0, 1, 2, 3, 4, ...$ in $[1 + n(n + 1) \times 3]$.

Cube Root by Prime Factorisation

Let us find the cube root of 74088.

Resolve 74088 into prime factors.

$$74088 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$$

$$\therefore \quad \sqrt[3]{74088} = 2 \times 3 \times 7 = 42$$

The steps involved to find the cube root by prime factorisation are as under:

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

Step 1: Resolve the given perfect cube into its prime factors.

Step 2: Group the factors into triplets.

Step 3: Take one factor out of each triplet.

Step 4: Multiply all the factors taken from the triplets. Their product will be the required cube root.

Let us follow the given rule to find cube root of some numbers through prime factorisation.

Example 5: Find the cube root of – 2744.

Solution: Resolve 2744 into prime factors.

2	2744
2	1372
2	686
2	343
7	49
7	7
	1

$$2744 = \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}$$

$$\therefore \quad \sqrt[3]{2744} = 2 \times 7 = 14$$

Hence, $\sqrt[3]{-2744} = -\sqrt[3]{2744} = -14$

Thus, from the above example we observe that—

For a positive integer x,

$$-x = -1 \times x$$

$$\therefore \quad \sqrt[3]{-x} = \sqrt[3]{(-1)\times x} = \sqrt[3]{-1} \times \sqrt[3]{x}$$

$\because \sqrt[3]{a \times b} = \sqrt[3]{a} \times \sqrt[3]{b}$

However, $\sqrt[3]{-1} = -1$ as $(-1)^3 = -1 \times -1 \times -1 = -1$

$$\therefore \quad \sqrt[3]{-x} = -\sqrt[3]{x}$$

Example 6: Find the cube root of – 3375.

Solution: We have, $\sqrt[3]{-3375} = -\sqrt[3]{3375}$

5	3375
5	575
5	135
3	27
3	9
3	3
	1

Resolve 3375 into prime factors.

$$3375 = \underline{5 \times 5 \times 5} \times \underline{3 \times 3 \times 3}$$

$$\sqrt[3]{3375} = 5 \times 3 = 15$$

$$\therefore \quad \sqrt[3]{-3375} = -15$$

Cube Root of Rational Numbers

Now, let us find $\sqrt[3]{\dfrac{8}{125}}$

$$\sqrt[3]{\frac{8}{125}} = \sqrt[3]{\frac{2\times2\times2}{5\times5\times5}} = \sqrt[3]{\left(\frac{2}{5}\right)^3}$$

or $$\sqrt[3]{\frac{8}{125}} = \sqrt[3]{\left(\frac{2}{5}\right)^3} = \frac{2}{5}$$

$$\frac{\sqrt[3]{8}}{\sqrt[3]{125}} = \frac{\sqrt[3]{2^3}}{\sqrt[3]{5^3}}$$

$$= \frac{2}{5}$$

Thus, if x and y (where $y \neq 0$) are perfect cubes,

Then, $$\sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}}$$

Example 7: Find the cube root of $\dfrac{729}{1000}$.

Solution: $\sqrt[3]{\dfrac{729}{1000}} = \dfrac{\sqrt[3]{729}}{\sqrt[3]{1000}}$

3	729		2	1000
3	243		2	500
3	81		2	250
3	27		5	125
3	9		5	25
3	3		5	5
	1			1

Resolving 729 and 1000 into prime factors.

$$729 = \underline{3\times3\times3}\times\underline{3\times3\times3}$$

$$1000 = \underline{2\times2\times2}\times\underline{5\times5\times5}$$

$\therefore$ $$\sqrt[3]{729} = 3\times3 = 9$$

and $$\sqrt[3]{1000} = 2\times5 = 10$$

Hence, $$\sqrt[3]{\frac{729}{1000}} = \frac{\sqrt[3]{729}}{\sqrt[3]{1000}} = \frac{9}{10}$$

Cube Root of a Number through Estimation

Let us find the cube root of a given number through estimation.

Consider the number 13824.

Step 1: Start making the groups of three digits starting from the right most digit of the number,

i.e.	Second group	First group
	13	824

Step 2: From the first group 824, take the digit from ones place. This is 4, which will be the ones digit in the cube root of the given number (as $4^3 = \underline{64}$ so 4 in the ones place of the required cube root).

Step 3: Now, take other group, i.e. 13.

We know that $8 < 13 < 27,$

i.e. $2^3 < 13 < 3^3$

So we take the ones place of the smaller number, i.e. 2 as the tens place of the required cube root.

So $\sqrt[3]{13824} = 24$

Example 8: Find the cube root of 175616 through estimation.

Solution:

Step 1: Form the groups of three digits starting from the right most digit of 175616.

II	I
175	616

Step 2: Let us consider the first group, i.e. 616. It has 6 in its ones place. Now, $6^3 = 216$ so 216 also has 6 in its place. So this gives the number at ones place of the required cube root.

Step 3: Now, consider the second group, i.e. 175

Now, $125 < 175 < 216,$

i.e. $5^3 < 175 < 6^3$

So the smaller number between 5 and 6 is 5 which qualifies for the tens place of the cube root.

$\therefore \quad \sqrt[3]{175616} = 56$

Worksheet 2

1. Find the cube root of the following by prime factorisation.

(i) 5832 (ii) 1728 (iii) 216000 (iv) 21952

2. Find the cube root of the following integers.

(i) – 1728 (ii) – 2744000 (iii) – 474552 (iv) – 5832

3. **Evaluate:**

(i) $\sqrt[3]{8\times125}$ (ii) $\sqrt[3]{3375\times(-729)}$ (iii) $\sqrt[3]{4^3\times5^3}$

4. **Find the cube root of the following rational numbers.**

(i) $\frac{4913}{3375}$ (ii) $\frac{-512}{343}$ (iii) $\frac{-686}{-2662}$

5. **By which smallest number must 5400 be multiplied to make it a perfect cube?**

6. **Find the smallest number by which 16384 be divided so that the quotient may be a perfect cube.**

7. **Find the cube root of the following numbers through estimation.**

(i) 10648 (ii) 15625 (iii) 110592 (iv) 91125

Value Based Questions

1. **Students of a school collected provisions like rice, pulses, etc., for the flood affected people of Madhya Pradesh. These provisions were packed in six cubical cartons each of side measuring 65 cm.**

   (a) Find the volume of cartons packed.

   (b) What values do you learn from the students?

2. **A school decided to award prizes to students for three values—discipline, cleanliness of environment and regularity in attendance. The number of students getting prizes in the three categories are in the ratio 1 : 2 : 3. If product of ratios is 162, then—**

   (a) Find the number of students getting prizes for each value.

   (b) Name any other two values that you can inculcate.

Brain Teasers

1.A. Tick (✓) the correct option.

(a) Cube of 0.1 is equal to—

(i) 1.11 (ii) 0.001 (iii) 0.101 (iv) 0.01

(b) The smallest number by which 1944 should be multiplied so that it becomes a perfect cube is—

(i) 3 (ii) 2 (iii) 5 (iv) 4

(c) Value of $\sqrt[3]{1000000}$ is—

(i) 10 (ii) 1000 (iii) 100 (iv) none of these

(d) $\sqrt[3]{0.027} - \sqrt[3]{0.008}$ is equal to—

(i) 1 (ii) 0.1 (iii) 0.11 (iv) 0.09

(e) Cube of $\left(\frac{-1}{3}\right)$ is equal to—

(i) $\frac{1}{27}$ (ii) $\frac{-1}{9}$ (iii) $\frac{-1}{27}$ (iv) $\frac{1}{9}$

B. Answer the following questions.

(a) Find the number whose cube is 1728.

(b) Find the value of $\sqrt[3]{216 \times (-125)}$.

(c) Find the cube root of 0.000001

(d) What is the smallest number by which 1715 should be divided so that the quotient is a perfect cube?

(e) Evaluate : $\sqrt[3]{\frac{0.512}{0.343}}$

2. Prove that if a number is tripled, then its cube is 27 times the cube of the given number.

3. Write cubes of all natural numbers between 1 to 10 and observe the pattern.

4. Find the cubes of:

(i) 0.6 (ii) – 3.1 (iii) – 0.01

5. Find the value of the following cube roots.

(i) $\sqrt[3]{0.008}$ (ii) $\sqrt[3]{\frac{-64}{1331}}$ (iii) $\sqrt[3]{27 \times 2744}$

6. **Find the smallest number which when multiplied with 3600 will make the product a perfect cube. Further, find the cube root of the product.**

7. **Evaluate:** $\sqrt[3]{\frac{0.027}{0.008}} \div \sqrt{\frac{0.09}{0.04}} - 1$

8. **Guess the cube root of the following numbers.**

   (i) 6859 (ii) 12167 (iii) 32768

HOTS

1. **Evaluate :** $\sqrt[3]{288\sqrt[3]{72\sqrt[3]{27}}}$

2. **Three numbers are in the ratio 2 : 3 : 4. The sum of their cubes is 33957. Find the numbers.**

Enrichment Questions

1. **Find the cube root of 4741632 by estimation.**

2. **Find the volume of a cube whose surface area is 150 m^2.**

You Must Know

1. The cube of a number is the number raised to the power 3.

2. The cube of an even natural number is even.

3. The cube of an odd natural number is odd.

4. The cube root of a number x is the number whose cube is x. It is denoted by $\sqrt[3]{x}$.

5. For any positive integer x, we have = $\sqrt[3]{-x} = -\sqrt[3]{x}$.

6. For any two integers a and b, we have,

   (i) $\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$.

   (ii) $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}, b \neq 0$.