STUDY MATERIAL OF MATHEMATICS : CLASS- XII

TOPIC : CONTINUITY

Definition of Limit : If x approaches a, i.e. $x \to a$, then f(x) approaches l, i.e. $f(x) \to l$, where l is a real number , then l is called limit of the function f(x) . Symbolically , $\lim_{x \to a} f(x) = l$.

Left Hand Limit : $\lim_{x \to a^-} f(x)$ is the expected value of f at x = a given the value of f near x to the left of a i.e.,

LHL = $\lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a - h)$, where h is very small and h >0.

Right Hand Limit : $\lim_{x \to a^+} f(x)$ is the expected value of f at x = a given the value of f near x to the right of a i.e.,

RHL = $\lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a + h)$, where h is very small and h >0.

Existence Of Limit : Limit will exist if LHL and RHL both exist i.e. finite and unique .

(ii) LHL (at $x = a$) = RHL (at $x = a$) $\neq f(a)$

Continuity in an interval : A function $y = f(x)$ is said to be continuous in an interval (a,b) iff $f(x)$ is continuous at every point in that interval and f is said to be continuous in the interval $[a,b]$ iff f is continuous in the interval (a,b) and also at the point a from the right and at the point b from the left .

Useful results for continuity :

(i) Every identity function is continuous .
(ii) Every constant function is continuous .
(iii) Every polynomial function is continuous .
(iv) Every rational function is continuous .
(v) All trigonometric functions are continuous in their domain .
(vi) Modulus function is continuous .

Algebra of continuous functions :

Suppose f and g are two real functions , continuous at real number a . Then ,

(i) $f + g$ is continuous at $x = a$.

Example : Evaluate the left hand and right hand limits of the function at x = 2.

f(x) = $\begin{cases} 2x+3 & if\ x \le 2 \\ x+5 & if\ x > 2 \end{cases}$. Does $\lim_{x\to 2} f(x)$ exist ?

Solution : LHL = $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(2x+3)$

$= \lim_{h\to 0}[2(2-h)+3] = 7$.

RHL = $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x+5)$

$= \lim_{h\to 0}(2+h+5) = 7$.

Hence LHL = RHL .

Therefore , limit exists and is equal to 7 .

Continuity at a point : A function f(x) is said to be continuous at a point x = a , if LHL (at x = a) = RHL (at x = a) = f(a) or $\lim_{x\to a} f(x) = f(a)$.

Discontinuity of a function : A function f(x) is said to be discontinuous at x = a when any of the following cases arise :

(i) LHL (at x = a) ≠ RHL (at x = a)

(ii) f − g is continuous at x = a .
(iii) f .g is continuous at x = a .
(iv) kf is continuous at x = a , where k is any constant .
(v) $\left(\frac{f}{g}\right)$ is continuous at x = a [provided g(a) ≠ 0] .

Example 1 : Discuss the continuity of f(x) = 5x – 3 at x = - 3 .

Solution : LHL (at x = - 3) = $\lim_{h\to 0}[5(-3-h)-3] = -18$.

RHL (at x = - 3) = $\lim_{h\to 0}[5(-3+h)-3] = -18$.

And f(- 3) = - 18 .

So LHL = RHL = f(- 3) = - 18 .

Hence , f(x) is continuous at x = - 3 .

Example 2 : Discuss the continuity of f(x) =

$$\begin{cases} \frac{\sin 2x}{\sin 3x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$$

at x = 0 .

Tools

At $x = 2$, RHL $= 2a + b$ and f(2)

As function is continuous at x

$= 5$ ....(1)

Now at $x = 10$, LHL $= 10a + b$ a

Therefore , $10a + b = 21$ ....(2)

Solving (1) and (2) we get , a = 2

PRACTICE PAPER : NCERT EXERCISE

Solution : $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x\to 0} \left(\frac{\sin 2x}{2x} \times \frac{3x}{\sin 3x} \times \frac{2}{3}\right)$.

$= \frac{2}{3} \times 1 \times 1 = \frac{2}{3}$.

Also f(0) = 0 .

$\therefore \lim_{x\to 0} f(x) \neq f(0)$.

So , f(x) is discontinuous at x = 0 .

Example 3 : Discuss the continuity of the function f ,

where f is defined by $f(x) = \begin{cases} -2 & if\ x \le -1 \\ 2x & if\ -1 < x \le 1 \\ 2 & if\ x > 1 \end{cases}$.

Solution : Here we discuss the continuity of the function at

x = - 1 and 1 .

At x = - 1, LHL = - 2 , RHL = - 2 and f(- 1) = - 2 .

Therefore , f(x) is continuous at x = - 1 .

At x = 1 , LHL = 2 , RHL = 2 and f(1) = 2 .

Therefore , f(x) is continuous at x = 1 .

Hence , f(x) is continuous for every value of x i

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Example 4 : Find the value of k , when the f

is continuous at x = 0 where f(x) = $\begin{cases} \frac{\sin x}{x} + \cos x \\ k \end{cases}$

Solution : Since function is continuous, so

f(0) .

Here RHL = 2 and f(0) = k .

Therefore , k = 2 .

Example 5 : Find the values of a and b such

function is defined by f(x) = $\begin{cases} 5 & if\ x \leq 2 \\ ax + b & if\ 2 < x < \end{cases}$

ASSIGNMENT

1 Mark Questions

1. Determine the value of 'k' for which the following function is continuous at $x = 3$:

$$f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

All India 2017

2. Determine the value of the constant 'k' so that the function $f(x) = \begin{cases} \dfrac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$ is continuous at $x = 0$. **Delhi 2017**

4 Marks Questions

3. Find the values of p and q for which

$$f(x) = \begin{cases} \dfrac{1 - \sin^3 x}{3\cos^2 x}, & \text{if } x < \dfrac{\pi}{2} \\ p, & \text{if } x = \dfrac{\pi}{2} \\ \dfrac{q(1 - \sin x)}{(\pi - 2x)^2}, & \text{if } x > \dfrac{\pi}{2} \end{cases}$$

is continuous at $x = \dfrac{\pi}{2}$. **Delhi 2016**

4. If $f(x) = \begin{cases} \dfrac{\sin(a+1)x + 2\sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \dfrac{\sqrt{1+bx} - 1}{\ldots}, & x > 0 \end{cases}$

6. If $f(x) = \begin{cases} \dfrac{1 - \cos 4x}{x^2}, & \ldots \\ a, & \ldots \\ \dfrac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & \ldots \end{cases}$

and f is continuous at $x =$ … value of a. **Delhi 2013C**

7. Find the value of k, for w…

$$f(x) = \begin{cases} \dfrac{\sqrt{1+kx} - \sqrt{1-\ldots}}{x}, & \ldots \\ \dfrac{2x+1}{x-1}, & \ldots \end{cases}$$

is continuous at $x = 0$. **All …**

8. Find the value of k, so th… function is continuous a…

$$f(x) = \begin{cases} \dfrac{x^3 + x^2 - 16x \ldots}{(x-2)^2} \\ k, \end{cases}$$

Delhi 2012C

9. Find the value of k, so … f defined by

$$f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & \text{if } \ldots \\ 3, & \text{if } \ldots \end{cases}$$

is continuous at $x = \dfrac{\pi}{2}$ …

10. Find the value of a fo… f is defined as

$$f(x) = \begin{cases} a\sin\dfrac{\pi}{2}(x+1) \ldots \\ \tan x - \sin x \ldots \end{cases}$$