Solutions

1. Capacitance of the capacitor with vacuum between its plates is $C_0 = \frac{\varepsilon_0 A}{d}$. [1/2]

When a sheet of relative permittivity K and thickness t is placed between the plates of a capacitor, its capacitance becomes,

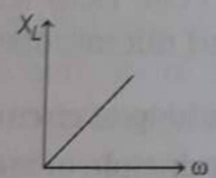
$$C = \frac{KC_0}{\left[\frac{t}{d} + K\left(1 - \frac{t}{d}\right)\right]}$$

As, $t = d$

Therefore, $C = KC_0$ [1/2]

2. (b) Inductive reactance, $X_L = \omega L \Rightarrow X_L \propto \omega$

Hence, inductive reactance increases linearly with angular frequency. So, the graph is correctly shown as



[1]

Or

The self-inductance of a circular coil, $L = \mu_0 n^2 Al$

$$L \propto n^2$$ [1/2]

So, if n is double, then self-inductance will be four times. [1/2]

3. As, $I = \frac{E}{R + r}$ or $E = I(R + r)$

$\Rightarrow \quad 4 = 0.2\,(10 + r)$

$\Rightarrow \quad 10 + r = \frac{4}{2} \times 10$

$\Rightarrow \quad r = 20 - 10 = 10\,\Omega$ [1]

4. The transition from $n = 4$ to $n = 2$ emits second line of Balmer series. [1]

5. Energy released by a positron and an electron,

$$E = mc^2 = 9.1 \times 10^{-31} \times (3 \times 10^8)^2 \text{ J}$$

$$= 81.9 \times 10^{-15}\text{ J} = 8.19 \times 10^{-14}\text{ J}$$ [1]

Or

Here, $R = 3 \times 10^{-15}$m

Nuclear mass = 16 amu

$$= 16 \times 1.66 \times 10^{-27}\text{ kg}$$

$$\rho_{nuclear} = \frac{\text{nuclear mass}}{\text{nuclear volume}} = \frac{16 \times 1.66 \times 10^{-27}}{\frac{4}{3}\pi (3 \times 10^{-15})^3}$$

$$= 2.359 \times 10^{17}\text{ kg m}^{-3}$$ [1]

6. Angular magnification is given as

$$m = \left(1 - \frac{v_0}{f_0}\right)\left(1 + \frac{D}{f_e}\right)$$ [1/2]

where, v_0 is the image distance for objective, f_e & f_0 are the focal lengths of eyepiece & objective lens, respectively and D is the least distance of distinct vision. [1/2]

7. As we know, magnitude of induced emf,

$$e = \frac{d\phi}{dt} = \frac{d}{dt}(3t^2 + 4t + 9) = 6t + 4 + 0$$ [1/2]

At $t = 2s$,

$$e = 6 \times 2 + 4 = 16 \text{ V}$$ [1/2]

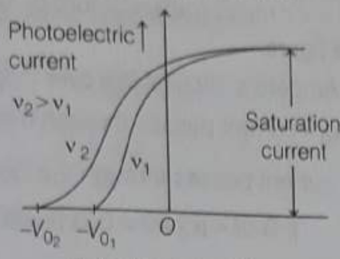
Or

Given, $V_0 = 120$ V

The rms value of voltage,

$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{120}{1.414} = 84.8 \text{ V}$$ [1]

8. Variation of photoelectric current *versus* potential for different frequencies and constant intensity of incident radiation is as shown in figure below [1/2]



The value of stopping potential is more negative for incident radiation of higher frequency. Hence, $\nu_2 > \nu_1$. [1/2]

9. Let n_h and n_e are the number of holes and number of conduction electrons respectively, in a semiconductor. [1/2]

Then,

(i) for intrinsic semiconductor, $n_e = n_h$

(ii) for n-type semiconductor, $n_e >> n_h$ [1/2]

Or

No, because there are no free charge carriers in the depletion region. Hence, there is no current in the circuit in the absence of any internal battery. [1]

10. We know that, $\mu = \frac{1}{\sin C}$

$$\Rightarrow \quad \sin C = \frac{1}{\mu} = \frac{1}{2/\sqrt{3}} = \frac{\sqrt{3}}{2}$$ [1/2]

$$\text{or} \quad C = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$$ [1/2]

11. (d) Current sensitivity, $S_i = \frac{NBA}{k}$

$$S_i \propto N$$

and voltage sensitivity, $S_V = \frac{NBA}{kR}$

$$S_V \propto \frac{N}{R}$$

So, when S_i is increased by increasing number of turns N, length of wire used also increases and so resistance R increases.

Hence, S_V may remain same or decrease whereas S_i increases.

Therefore, A is false and R is also false. [1]

12. (a) If the inner solenoid is much shorter than (and placed well inside) the outer solenoid, then the flux linkage $N_1\phi_1$ can still be calculated.

It is because the inner solenoid is effectively immersed in a uniform magnetic field due to the outer solenoid.

Therefore, A and R are true and R is the correct explanation of A. [1]

13. (a) In optical fibre communication, propagation of signal through optical fibre takes place, which is based on the phenomenon of total internal reflection at core-clad interface.

The refractive index of the material of the cladding is less than that of core, hence light striking at core-cladding interface gets totally internally reflected.

Therefore, A and R are true and R is the correct explanation of A. [1]

14. (c) The conductivity of an intrinsic semiconductor is less than that of a slightly doped p-type semiconductor. Also, with the increase in the temperature, conductivity of intrinsic semiconductor increase.

Therefore, A is true but R is false. [1]

15. (i) (d) Total internal reflection is the basis for following phenomenon

(a) Sparkling of diamond.

(b) Optical fibre communication.

(c) Instrument used by doctors for endoscopy. [1]

(ii) (d) Total internal reflection (TIR) is the phenomenon that involves the reflection of all the incident light off the boundary. TIR only takes place when both of the following two conditions are met

The light is in the more denser medium and approaching the less denser medium.

The angle of incidence is greater than the so-called critical angle. [1]

(iii) (c) If incidence angle, i = critical angle C, then refraction angle, $r = 90°$. [1]

(iv) (b) In optical fibres, core is surrounded by cladding, where the refractive index of the material of the core is higher than that of cladding to bound the light rays inside the core. [1]

(v) (b) From Snell's law, $\sin C = {}_1n_2 = \frac{v_1}{v_2}$

where, C = critical angle = 30°

and v_1 & v_2 are speed of light in medium & vacuum, respectively.

We know that, $v_2 = 3 \times 10^8$ m/s

$$\therefore \quad \sin 30° = \frac{v_1}{3 \times 10^8}$$

$$\Rightarrow \quad v_1 = 3 \times 10^8 \times \frac{1}{2} \Rightarrow v_1 = 1.5 \times 10^8 \text{ m/s}$$ [1]

16. (i) (a) Since, $E = 0$ inside the conductor and has no tangential component on the surface, no work is done in moving a small test charge within the conductor and on its surface. [1]

(ii) (c) The positively charged particle experiences electrostatic force along the direction of electric field, i.e. from high electrostatic potential to low electrostatic potential. Thus, the work is done by the electric field on the positive charge, hence electrostatic potential energy of the positive charge decreases. [1]

(iii) (d) Potential energy of the system,

$$U = \frac{KQq}{l} + \frac{Kq^2}{l} + \frac{KqQ}{l} = 0$$

$$\Rightarrow \quad \frac{Kq}{l} \times [(Q + q + Q)] = 0$$

$$\Rightarrow \quad Q = -q/2$$ [1]

(iv) (a) Since, the proton is moving against the direction of electric field, so work is done on the proton against electric field. It implies that electric field does negative work on the proton. Again, proton is moving in electric field from low potential region to high potential region hence, its potential energy increases. [1]

(v) (c) Electric potential energy of the system,

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

Here, $q_1 = q_2 = 1\mu C = 1 \times 10^{-6} C$,

$r = 1\,m$ and $\frac{1}{4\pi\varepsilon_0}$

$$= 9 \times 10^9 N\text{-}m^2/C^2$$

$$\therefore \quad U = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 1 \times 10^{-6}}{1}$$

$$= 9 \times 10^{-3} J$$ [1]

17. Mobility is defined as the magnitude of drift velocity of charges per unit electric field applied. [1]

It is expressed as

$$\mu = \frac{\text{Drift velocity } (v_d)}{\text{Electric field } (E)} = \frac{v_d}{E}$$

Since, drift velocity can also be given as

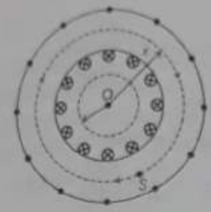
$$v_d = \frac{qE\tau}{m}$$

where, τ is relaxation time.

$$\Rightarrow \quad \mu = \frac{\frac{qE\tau}{m}}{E} = \frac{q\tau}{m}$$

Its SI unit is $m^2s^{-1}V^{-1}$ or $ms^{-1}N^{-1}C$. [1]

18. Toroid is a hollow circular ring on which a large number of insulated turns of a metallic wire are closely wound.



Let B is the magnetic field in the open space interior to the toroid. [1]

Considering a loop coplanar with toroid of radius x such that $x < r$ mean (radius of toroid) as shown in the above figure.

Applying Ampere's circuital law over loop, we have $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ current passes through the loop.

Since, no current passes through the loop.

$$\therefore \quad \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times 0 = 0 \Rightarrow B = 0$$

Thus, the magnetic field exists in the open space, however in the interior of toroid, it is zero. [1]

19. Number of atoms present in 239 g of $^{239}_{94}Pu = 6.023 \times 10^{23}$.

$\therefore$ Number of atoms present in 1 kg or 1000 g of $^{239}_{94}Pu$

$$= \frac{6.023 \times 10^{23} \times 1000}{239} = 2.52 \times 10^{24}$$ [1]

Energy released per fission = 180 MeV

Total energy released $= 2.52 \times 10^{24} \times 180$ MeV

$$= 4.54 \times 10^{26} \text{ MeV}$$ [1]

Or

According to Bohr's theory, centripetal force required by the electron for its motion around the nucleus = electric force between the proton and electron.

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(q_p)(q_e)}{r^2}$$ [from Coulomb's law]

where, r = atomic radius, q_p = charge of proton = $+e$ and q_e = charge of electron = $-e$.

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(e)(-e)}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-e^2}{r^2}$$ [1]

Now, given charge on proton, $q_p = +\frac{4}{3}e$

Charge on electron, $q_e = -\frac{3}{4}e$

Putting the new value (keeping after factors unchanged),

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\left(\frac{4}{3}e\right)\left(-\frac{3}{4}e\right)}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-e^2}{r^2}$$

i.e. Bohr's formula remains unchanged. [1]

20. In a single slit diffraction pattern, the angular width of central maxima, $2\theta = 2\lambda/a$ ...(i)

where, λ is wavelength and a is width of slit.

Therefore, [1]

(i) when the slit width is decreased, then from Eq. (i), angular width of central bright maximum increases.

(ii) when the light of smaller wavelength λ is used, then angular width of central bright maximum decreases. [1]

Or

Let f and $-f$ be the focal lengths of the converging and diverging lenses, respectively.

As, power, $P = \frac{1}{\text{focal length, } f}$

So, $$P_{net} = P_1 + P_2 = \frac{1}{f} + \frac{1}{-f} = 0$$ [1]

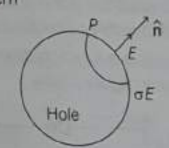
Thus, focal length of the combination,

$$f = \frac{1}{P} = \frac{1}{0} = \infty$$ [1]

21. (i) 0.1 m to 1 mm corresponds to microwaves. They are used in RADAR systems for aircraft navigation. [1]

(ii) 1 nm to 10^{-3} nm corresponds to X-rays. They are used in surgeries to detect fracture, diseased organs, stones in body, etc. [1]

22. Let P be the point on the hole. The electric field at point P closed to the surface to conductor, according to Gauss' theorem



$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

where, q is charge near the hole.

Since, angle between electric field and area vector is 0°.

$$\therefore \quad E \cdot dS = \frac{\sigma dS}{\varepsilon_0} \quad (\because q = \sigma dS)$$

$$E = \frac{\sigma}{\varepsilon_0} \Rightarrow \mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$ [1]

where, $\hat{n}$ is unit vector in normal direction.

This electric field is due to the filled up hole and the field due to the rest of charged conductor. The two fields inside the conductor are equal and opposite, so there is no electric field inside the conductor. Outside the conductor, the electric fields are equal in the same direction.

So, electric field at point P due to each part

$$= \frac{1}{2}E = \frac{\sigma}{2\varepsilon_0}\hat{n}$$ [1]

23. Given, wavelength, $\lambda = 6000$ nm $= 6000 \times 10^{-9}$ m

Energy of incident photon, $E = h\nu = \frac{hc}{\lambda}$ [1/2]

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{6000 \times 10^{-9}} J$$

$$= \frac{3.3 \times 10^{-20}}{1.6 \times 10^{-19}} eV$$

$$\Rightarrow \quad E = 0.207 \text{ eV} \quad \text{...(i)}$$ [1/2]

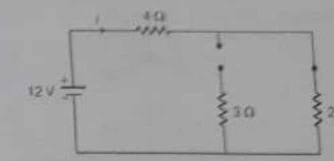
Energy required to cross the gap,

$$E_g = 2.8 \text{ eV} \quad \text{...(ii)}$$

$\because E < E_g$, the p-n junction cannot detect the given wavelength of 6000 nm. [1]

Or

Here, D_1 is reverse biased and D_2 is forward biased. So, the circuit can be redrawn as [1/2]



[1/2]

Net resistance of the circuit, $R = 4\Omega + 2\Omega = 6\Omega$

Voltage = 12 V [1/2]

$\therefore$ The current flowing, $I = \frac{V}{R} = \frac{12}{6} = 2A$ [1/2]

24. (i) Work function, $\phi = h\nu_0 \Rightarrow \phi \propto \nu_0$

Higher the threshold frequency

= Higher the work function

According to the graph given in the question,

$$\nu_0' > \nu_0$$

$\therefore$ Metal A has higher work function (ϕ). [1]

(ii) Since, $KE_{max} = h\nu - \phi$

(Einstein's photoelectric equation)

$$\Rightarrow \quad eV_0 = h\nu - \phi$$

Dividing by e on both sides, we get

$$V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi}{e}$$

If we compare it with $y = mx + c$,

then we get $y = V_0$ and $m = \frac{h}{e}$, $x = \nu$ and $c = \frac{-\phi}{e}$

Thus, the slope of (V_0 versus ν) graph $= \frac{h}{e}$ = constant. [1]

25. Given, frequency of oscillation, $f = 2 \times 10^{10}$ Hz

and electric field amplitude, $E_0 = 48$ V/m

Since, we know that, $c = 3 \times 10^8$ m/s

(i) Wavelength of the waves,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{2 \times 10^{10}}$$

$$= 1.5 \times 10^{-2}\ m \qquad [1/2]$$

(ii) The average energy density of electric field,

$$u_E = \frac{1}{2}\varepsilon_0 E_0^2 \qquad \text{(i)}$$

We know that, $\frac{E_0}{B_0} = c$

Putting in Eq. (i), we get

$$\therefore \quad u_E = \frac{1}{2}\varepsilon_0 c^2 B_0^2 \qquad \text{(ii)}$$

Speed of electromagnetic waves,

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad \text{(iii)} \quad [1/2]$$

Putting Eq. (iii) in Eq. (ii), we get

$$u_E = \frac{1}{2}\varepsilon_0 B_0^2 \cdot \frac{1}{\mu_0 \varepsilon_0}$$

$$= \frac{1}{2} \cdot \frac{B_0^2}{\mu_0} = u_B$$

Thus, the average energy density of the **E** field equals the average energy density of **B** field. [1]

26. (i) Component of velocity perpendicular to the rod

$= v \sin \theta$

Therefore, in time t, area transversed (A)

$= x \times v \sin \theta \times t$

Since, $\phi = \mathbf{B} \cdot \mathbf{A} = BA \cos \theta$

Here, $\theta = 0°$

$\therefore \quad \phi = B(x \times v \sin \theta \times t) \cos 0° = Bxvt \sin \theta$

Also, $|e| = \frac{d\phi}{dt} = Bxv \sin \theta$ [1½]

(ii) In time t, if θ is the angle traced by the free end, then

area swept, $A = \pi x^2 \times \left(\frac{\theta}{2\pi}\right) = \frac{1}{2}x^2\theta$

Also, $\phi = B\left(\frac{1}{2}x^2\theta\right)\cos 0° = \frac{1}{2}Bx^2\theta$

$\therefore \quad |e| = \frac{d\phi}{dt}$

$$= \frac{1}{2}Bx^2\frac{d\theta}{dt} = \frac{1}{2}Bx^2\omega \qquad [1½]$$

Or

Given, diameter of solenoid is d and number of turns per length is n.

Magnetic field due to a solenoid, $\mathbf{B} = \mu_0 nI$

Magnetic field in smaller coil, $\phi = NBA$ (where, $A = \pi b^2$)

Induced emf appearing in the smaller coil

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA)$$

$$= -N \cdot \pi b^2 \frac{dB}{dt} = -N\pi b^2 \frac{d}{dt}(\mu_0 nI)$$

or $\quad e = -N\pi b^2 \mu_0 n \frac{dI}{dt}$

or $\quad e = -N\pi b^2 \mu_0 \frac{nd}{dt}(mt^2 + C)$

$[\because I = mt^2 + C(d) = mt^2 + C]$

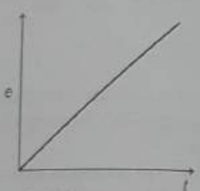
$$e = -\mu_0 Nn\pi b^2 \cdot 2mt$$

The negative sign shows that the opposite nature of induced emf by Lenz's law. [2]

Variation of emf (e) with time (t)

$$e = -\mu_0 Nn\pi b^2 \cdot 2mt$$

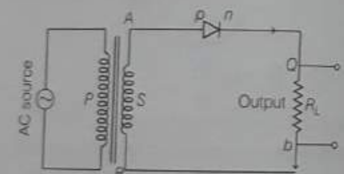
or $\quad e \propto t$



Graph between e and t [1]

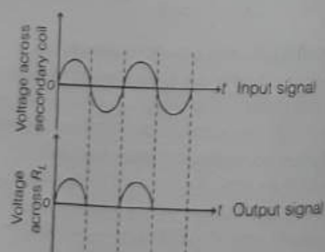
27. Principle of Rectifier If an alternating voltage is applied across a diode, the current only flows in that part of the cycle when the diode is forward biased. This property is used to rectify current/voltage. [1]

Circuit diagram for *p-n* junction diode as half-wave rectifier is shown in figure below



[1]

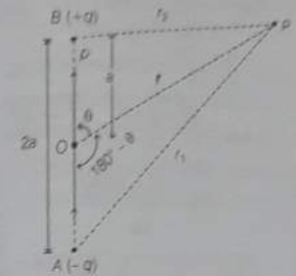
Sketch of the input and output waveforms are as shown in figure below



[1]

28. Let us consider an electric dipole consisting of charges $+q$ and $-q$ seperated by a distance $2a$

The dipole moment $|\mathbf{p}| = q \times 2a$



Let O be the centre of the dipole, P be any point near the electric dipole inclined at an angle θ as shown in the above figure.

Let P be the point at which electric potential is required.

Potential at P due to $-q$ charge, $V_1 = \frac{-q}{4\pi\varepsilon_0 r_1}$

Potential at P due to $+q$ charge, $V_2 = \frac{q}{4\pi\varepsilon_0 r_2}$ [1]

Therefore, potential at P due to the dipole

$$V_P = V_1 + V_2 = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \qquad \text{...(i)}$$

Now, by geometry,

$$r_1^2 = r^2 + a^2 + 2ar\cos\theta$$

Similarly, $r_2^2 = r^2 + a^2 + 2ar\cos(180° - \theta)$

or $\quad r_2^2 = r^2 + a^2 - 2ar\cos\theta \ [\because \cos(180° - \theta) = -\cos\theta]$

and $\quad r_1^2 = r^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)$

If $r >> a$, $\frac{a}{r}$ is small. Therefore, $\frac{a^2}{r^2}$ can be neglected.

$$\therefore \quad \frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{2a}{r}\cos\theta\right)^{-1/2}$$

Similarly, $\quad \frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-1/2}$ [1]

Putting these values in Eq. (i), we obtain

$$V_P = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}} - \frac{1}{r}\left(1 + \frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}}\right]$$

Using Binomial theorem, $[(1+x)^n = 1 + nx, x << 1]$ and retaining terms upto the first order in $\frac{a}{r}$, we get

$$V_P = \frac{q}{4\pi\varepsilon_0 r}\left[\left(1 + \frac{a}{r}\cos\theta\right) - \left(1 - \frac{a}{r}\cos\theta\right)\right]$$

$$= \frac{q \times 2a\cos\theta}{4\pi\varepsilon_0 r^2}$$

$\Rightarrow \quad V_P = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \qquad [\because p = q \times 2a]$ [1]

Or

(i) Electric field at the location of wire 2 due to charge on wire 1 is

$$E = \frac{\lambda_1}{2\pi\varepsilon_0 d}$$

Force per unit length of wire 2 due to the above field

$F = E \times$ charge on the unit length of wire $2 = E\lambda_2$

$\Rightarrow \quad F = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0 d}$ [1½]

(ii) At any axial point of a dipole, electric field varies as

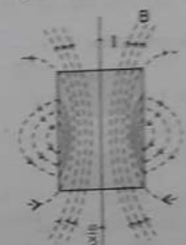
$$E \propto \frac{1}{r^3}$$

or $\quad \frac{F}{q} \propto \frac{1}{r^3}$

$\Rightarrow \quad F \propto \frac{1}{r^3}$

$\therefore$ When the distance of the point charge is doubled then the force reduceds to $F/8$ [1½]

29. (i) The magnetic field lines for a solenoid are as shown in figure below



[1]

(ii) The tangent to the field line at a given point represents the direction of the net magnetic field at that point. [1]

(iii) No, these lines are unlike the electric field lines of an electric dipole. Basically, electric field lines begins from a positive charge and end on the negative charge or escape to infinity.

Also, unlike magnetic field lines they do not form continuous loop. [1]

30. Yes, de-Broglie hypothesis can be used to derive the Bohr's second postulate. [1/2]

According to de-Broglie, a stationary orbit is that which contains an integral number of de-Broglie standing waves associated with the revolving electron.

For an electron revolving in nth circular orbit of radius r_n, total distance covered = circumference of the orbit $= 2\pi r_n$

$\therefore$ For the permissible orbit, $2\pi r_n = n\lambda$

According to de-Broglie wavelength, $\lambda = \dfrac{h}{mv_n}$

where, v_n is speed of electron revolving in nth orbit.

$$\therefore \quad 2\pi r_n = \frac{nh}{mv_n}$$

or $$mv_n r_n = \frac{nh}{2\pi} = n(h/2\pi)$$

i.e. angular momentum of electron revolving in nth orbit must be an integral multiple of $h/2\pi$, which is the quantum condition proposed by Bohr in his second postulate. [2½]

31. (i) The alternating emf in the above L-C-R series circuit would be represented by

$$V = V_0 \sin(1000t + \phi) \Rightarrow \omega = 1000 \text{ Hz}$$

Given, $R = 400\,\Omega$, $C = 2\,\mu\text{F} = 2 \times 10^{-6}$ F

and $L = 100$ mH $= 100 \times 10^{-3}$ H

$\because$ Capacitive reactance, $X_C = \dfrac{1}{\omega C}$

$$\Rightarrow \quad X_C = \frac{1}{1000 \times 2 \times 10^{-6}}$$

$$\Rightarrow \quad X_C = \frac{10^3}{2}$$

$$\Rightarrow \quad X_C = 500\,\Omega \quad [1]$$

$\because$ Inductive reactance, $X_L = \omega L$

$$\Rightarrow \quad X_L = 1000 \times 100 \times 10^{-3} = 100\,\Omega$$

So, we can see that $X_C > X_L$

$\tan\phi$ is negative.

Hence, the voltage lags behind the current by a phase angle ϕ. The AC circuit is capacitance dominated circuit. [1½]

$\because$ Phase difference, $\tan\phi = \dfrac{X_L - X_C}{R}$

$$\tan\phi = \frac{100 - 500}{400} \Rightarrow \tan\phi = \frac{-400}{400}$$

$$\tan\phi = -1$$

$$\Rightarrow \quad \tan\phi = -\tan\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \quad \phi = -\frac{\pi}{4}$$

This is the required value of the phase difference between the current and the voltage in the given series L-C-R circuit.

(ii) From the given graph, we get

$$I_1 = 1\text{A}$$
$$I_2 = -2\text{A}$$
$$I_3 = 1\text{A}$$

$$I_{rms} = \sqrt{\frac{I_1^2 + I_2^2 + I_3^2}{3}} = \sqrt{\frac{1^2 + (-2)^2 + 1^2}{3}}$$

$$= \sqrt{\frac{6}{3}} = \sqrt{2} = 1.414 \text{ A}$$

Or

(i) As we know that, $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$

Now, when capacitor is removed, $\tan\phi = \dfrac{\omega L}{R}$

and when inductor is removed, $\tan\phi = \dfrac{-\dfrac{1}{\omega C}}{R}$

Negative sign indicates that current leads the voltage.

$$\therefore \quad \omega L = 1/\omega C \Rightarrow \omega = 1/\sqrt{LC}$$

$\Rightarrow$ L-C-R circuit is in resonance.

$$\therefore \quad I_{rms} = V_{eff}/R = V/R$$

Thus, average power dissipated in the L-C-R circuit is

$$P_{av} = V_{rms} I_{rms} = V^2_{rms} \text{ in } R = \frac{V^2}{R}$$

(ii) Given, $V = 140 \sin(314t)$ and $R = 50\,\Omega$

Comparing it with $V = V_0 \sin\omega t$

(a) Here, $\omega = 314$ rad/s

i.e. $2\pi v = 314$ $[\because \omega = 2\pi v]$

$$\Rightarrow \quad v = \frac{314}{2\pi} = \frac{31400}{2 \times 314} = 50 \text{ Hz}$$

Frequency of AC, $v = 50$ Hz

(b) As, $I_{rms} = \dfrac{V_{rms}}{R}$ and $V_{rms} = \dfrac{V_0}{\sqrt{2}}$

Here, $V_0 = 140$ V

$$\Rightarrow \quad V_{rms} = \frac{140}{\sqrt{2}} = 70\sqrt{2} \text{ V}$$

$$\therefore \quad I_{rms} = \frac{70\sqrt{2}}{R} = \frac{70\sqrt{2}}{50} = 1.9\text{A or } 2\text{A}$$

32. (i) Diffraction of light occurs when size of the obstacle or the aperture is comparable to the wavelength of light.

Angular separation between diffraction fringes

$$\theta = \frac{\beta}{D} = \frac{\lambda}{d}$$

When the distance (D) between the slit and screen is doubled, the angular separation θ remains unchanged. [1]

(ii) Given, separation between two slits,

$$d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

Distance of screen from slit, $D = 120$ cm $= 1.2$ m

Wavelength, $\lambda_1 = 650$ nm $= 6.5 \times 10^{-7}$ m

Wavelength, $\lambda_2 = 520$ nm $= 5.2 \times 10^{-7}$ m

(a) Since, separation of nth order bright fringe from centre fringe is given by $Y_n = \dfrac{nD\lambda}{d}$

$\therefore$ For third order bright fringe, $Y_3 = \dfrac{3D\lambda_1}{d}$

where, $\lambda_1 = 6.5 \times 10^{-7}$ m

$$\therefore \quad Y_3 = \frac{3 \times 1.2 \times 6.5 \times 10^{-7}}{2 \times 10^{-3}}$$

$$= 1.17 \times 10^{-3} \text{m} = 1.17 \text{ mm} \quad [1]$$

(b) Let nth order bright fringe of wavelength λ_1 coincides with $(n+1)$th order bright fringe of wavelength λ_2.

$$\therefore \quad \frac{nD\lambda_1}{d} = \frac{(n+1)D\lambda_2}{d}$$

or $$n\lambda_1 = (n+1)\lambda_2$$

or $$\frac{n+1}{n} = \frac{\lambda_1}{\lambda_2} = \frac{6.5 \times 10^{-7}}{5.2 \times 10^{-7}}$$

or $$\left(1 + \frac{1}{n}\right) = \frac{5}{4} \text{ or } \frac{1}{n} = \frac{5}{4} - 1 = \frac{1}{4}$$

$$\Rightarrow \quad n = 4 \quad [1]$$

$\therefore$ The required least distance

$$= \frac{nD\lambda_1}{d} = \frac{4 \times 1.2 \times 6.5 \times 10^{-7}}{2 \times 10^{-3}}$$

$$= 1.56 \times 10^{-3} \text{ m}$$

$$= 1.56 \text{ mm} \quad [1]$$

Or

Given, focal length of objective, $f_o = 1.25$ cm,

focal length of eyepiece, $f_e = 5$ cm,

least distance of distinct vision, $D = 25$ cm

and angular magnification of lens, $M = 30$

The magnification produced by eyepiece,

$$M_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6 \quad [1]$$

The magnification produced by microscope,

$$M = M_o \times M_e$$

$$\Rightarrow \quad 30 = M_o \times 6$$

where, M_o is the magnification produced by objective lens.

$$M_o = 5 \quad [1]$$

Again, we know that magnification of objective lens,

$$\Rightarrow \quad 5 = \frac{-v_o}{u_o}$$

$$\Rightarrow \quad v_o = -5u_o \quad \text{...(i)}$$

Using lens formula for objective lens

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\Rightarrow \quad \frac{1}{1.25} = \frac{1}{-5u_o} - \frac{1}{u_o} = -\frac{6}{5u_o} \quad \text{[from Eq. (i)]}$$

$$u_o = -\frac{6}{5} \times 1.25 = -1.5 \text{ cm}$$

$$v_o = -5u_o$$

$$= -5(-1.5) = 7.5 \text{ cm}$$

Thus, the object should be placed at a distance of 1.5 cm from the objective lens to get the desired magnification. [1]

Now, using the lens formula for eyepiece

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

$$\Rightarrow \quad \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= -\frac{1}{25} - \frac{1}{5} = -\frac{6}{25} \quad [\because v_e = -25 \text{ cm}]$$

$$u_e = -4.17 \text{ cm} \quad [1]$$

The separation between objective and eyepiece

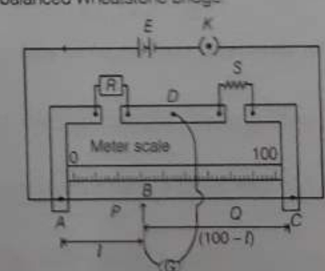
$$= |v_o| + |u_e| = 7.5 + 4.17$$

$$= 11.67 \text{ cm}$$

Thus, the microscope is settled as the distance between eyepiece and objective is 11.67 cm. [1]

33. (i) A meter bridge is also known as **slide wire bridge**. It is a practical form of Wheatstone bridge.

Principle It is constructed on the principle of balanced Wheatstone bridge.



At balancing situation of bridge,

$$\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{l}{100 - l} = \frac{R}{S}$$

$$\Rightarrow \quad S = \frac{100 - l}{l} \times R$$ [2]

(ii) Given, $E_2 = 1.02$ V, $l_1 = 1$ m $= 100$ cm and $l_2 = 51$ cm

(a) Potential gradient, $K = \frac{E_2}{l_2} = \frac{1.02\text{ V}}{51\text{ cm}} = 0.02\text{ V cm}^{-1}$ [1]

(b) $E_1 = Kl_1 = (0.02\text{ Vcm}^{-1}) \times 100\text{cm} \Rightarrow E_1 = 2\text{V}$ [1]

(c) No, when switch S is closed, position of the null point remains unaffected as no current flows through the cell (E_2) at null point. [1]

Or

(i) **First Law (Junction Rule)**

This law states that, the algebraic sum of the currents meeting at a point in an electrical circuit is always zero. It is also known as junction rule.

Sign convention for Kirchhoff's first law

The current flowing towards the junction of conductors is considered as positive and the current flowing away from the junction is taken as negative.

Second Law (Kirchhoff's Voltage Rule)

This law states that, the algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero. It means that in any closed part of an electrical circuit, the algebraic sum of the emf's is equal to the algebraic sum of the products of the resistances and currents flowing through them. It is also known as **loop rule**.

Sign convention for Kirchhoff 's second law

The product of resistance and current in an arm of the loop is taken as positive, if the direction of current in that arm is in the same sense as one moves and is taken as negative, if the direction of current in an arm is opposite to the sense as one moves.

While traversing a loop, the emf of a cell is taken negative, if negative pole of the cell is encountered first, otherwise positive. [2]

(ii) Applying Kirchhoff's second law to mesh $ABDA$,

$$-10I_1 - 5I_3 + 5I_2 = 0$$

$$2I_1 - I_2 + I_3 = 0 \quad \ldots\text{(i)}$$

Applying Kirchhoff's second law to mesh $BCDB$,

$$-5(I_1 - I_3) + 10(I_2 + I_3) + 5I_3 = 0$$

$$5I_1 - 10I_2 - 20I_3 = 0$$

$$I_1 - 2I_2 - 4I_3 = 0 \quad \ldots\text{(ii)}$$ [1/2]

Applying Kirchhoff's second law to mesh $ADCA$,

$$-5I_2 - 10(I_2 + I_3) + 10 - 10(I_1 + I_2) = 0$$

$$2I_1 + 5I_2 + 2I_3 = 2 \quad \ldots\text{(iii)}$$

From Eq. (i), $\quad I_2 = 2I_1 + I_3 \quad \ldots\text{(iv)}$

From Eq. (ii), $\quad I_1 = 2I_2 + 4I_3 \quad \ldots\text{(v)}$ [1/2]

$\therefore$ Substituting I_1 in Eq. (iv), we get

$$I_2 = 2(2I_2 + 4I_3) + I_3$$

$$I_2 = -3I_3$$

From Eq. (v)

$$I_1 = -6I_3 + 4I_3$$

$$\Rightarrow \quad I_1 = -2I_3$$

Now, from Eq. (iii)

$$-4I_3 - 15I_3 + 2I_3 = 2$$

$$\Rightarrow \quad I_3 = \frac{-2}{17}\text{ A}$$

$$\therefore \quad I_1 = \frac{+4}{17}\text{ A}, \; I_2 = \frac{6}{17}\text{ A}, \; I = I_1 + I_2 = \frac{10}{17}\text{ A}$$ [1]

So, current in branch $AB = I_1 = \frac{4}{17}$ A

Current in branch $BC = I_1 - I_3 = \frac{6}{17}$ A

Current in branch $AD = I_2 = \frac{6}{17}$ A

Current in branch $DC = I_2 + I_3 = \frac{4}{17}$ A

Current in branch $BD = I_3 = -\frac{2}{17}$ A

(Direction of current is from D to B)

Current in cell $= I_1 + I_2 = \frac{10}{17}$ A [1]