PRACTICE PAPER-III
SUB:- MATHEMATICS
CLASS - XII

Time : 3 hrs

Max. Marks : 80

General Instructions

1. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
2. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
3. Both Part A and Part B have choices.

PART – A

1. It consists of two sections- I and II.
2. Section I comprises of 16 Very Short Answer Type Questions.
3. Section II contains two Case Studies. Each Case Study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

PART – B

1. It consists of three sections- III, IV and V.
2. Section III comprises of 10 questions of 2 marks each.
3. Section IV comprises of 7 questions of 3 marks each.
4. Section V comprises of 3 questions of 5 marks each.
5. Internal choice is provided in 3 questions of Section –III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART A

Section I

All questions are compulsory. In case of internal choices attempt any one.

1. Find the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$.
2. Find the value of $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.
3. What is the range of the function $f(x) = \frac{|x-1|}{x-1}, x \neq 1$?
4. Write the name of the matrix obtained by interchanging the rows and columns of a given matrix A.

Or

If $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$, then find the cofactor A_{21}.

5. Find non-zero values of x satisfying the matrix equation.

$$x\begin{bmatrix}2x & 2\\ 3 & x\end{bmatrix}+2\begin{bmatrix}8 & 5x\\ 4 & 4x\end{bmatrix}=2\begin{bmatrix}x^2+8 & 24\\ 10 & 6x\end{bmatrix}.$$

6. If the matrix $A=\begin{bmatrix}0 & a & -3\\ 2 & 0 & -1\\ b & 1 & 0\end{bmatrix}$ is a skew-symmetric. Find the value of a and b.

Or

If the matrix $B=\begin{bmatrix}2 & a & 5\\ -1 & 4 & b\\ c & -4 & 9\end{bmatrix}$ is a symmetric matrix, then find $a+b+c$.

7. State whether $f(x)=\sin x+\cos x$ $x\in\left[0,\frac{\pi}{4}\right]$ is increasing or decreasing.

Or

Find the interval in which $y=x^2e^{-x}$ is increasing with respect to x

8. Evaluate : $\int x^x(1+\log x)\,dx$.

9. Find maximum and minimum value of function $h(x)=\sin(2x)+5$

Or

Find the maximum and minimum value of function $f(x)=|x+2|-1$.

10. If $P(A)=\frac{1}{2}$, $P(B)=0$, then find $P\left(\frac{A}{B}\right)$.

11. Let A and B be the events associated with the sample space S, then find the value of $P(A/B)$.

12. For any two vectors $\vec{a}$ and $\vec{b}$, if $\vec{a}\perp\vec{b}$, then find the value $\vec{a}\cdot\vec{b}$

Or

Write the value of projection vector of $\vec{a}$ along $\vec{b}$.

13. Find the distance of the point (2, 3, –5) from the plane $x+2y-2z-9=0$.

14. Find the distance between the planes $2x-y+2z=5$ and $5x-2.5y+5z=20$.

15. Find the vector equation of the plane with intercepts 3, –4, 2 on X, Y, Z-axis respectively.

16. If $\vec{a}=x\hat{i}+2\hat{j}-z\hat{k}$ and $\vec{b}=3\hat{i}-y\hat{j}+\hat{k}$ are two equal vectors, then write the value of $x+y+z$.

Section II

Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question. Each part carries 1 mark

17. The random variable X has a probability distribution $P(X)$ of the following form where k is some number:

$$P(X=x)=\begin{cases}k, & \text{if } x=0\\ 2k, & \text{if } x=1\\ 3k, & \text{if } x=2\\ 0, & \text{otherwise}\end{cases}$$

On the basis of above information answer the following questions:

(i) The value of k is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

(ii) $P(X=2)=$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

(iii) $P(X>2)=$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 0

(iv) $P(X<2)=$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

(v) $P(0<x<2)=$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

18. The sum of the surface areas of a rectangular parallelopiped with sides x, 2x and $\frac{x}{3}$ and a sphere of radius y is given to be constant.

On the basis of above information answer the following questions.

(i) Let the constant be S, then relation between x and y is

(a) $S=4x^2+6\pi y^2$

(b) $S=6x^2+4\pi y^2$

(c) $S=2x^2+3\pi y^2$

(d) $S=3x^2+2\pi y^2$

(ii) If the combined volume is denoted by V.

(a) $\frac{4}{3}\pi y^3+\frac{2}{3}x^3$ (b) $\frac{2}{3}\pi y^3+\frac{4}{3}x^3$

(c) $4\pi y^3+2x^3$ (d) $2\pi y^3+4x^3$

(iii) If V is minimum, then relation between x and y is

(a) $x=2y$ (b) $x=3y$

(c) $y=2x$ (d) $y=3x$

(iv) The value of minimum volume is

(a) $\frac{4}{3}x^3\left(1+\frac{2\pi}{27}\right)$ (b) $\frac{2}{3}x^3\left(1+\frac{4\pi}{27}\right)$

(c) $\frac{4}{3}x^3\left(1+\frac{4\pi}{27}\right)$ (d) $\frac{2}{3}x^3\left(1+\frac{2\pi}{27}\right)$

(v) The value of S, when V is minimum, is

(a) $2x^2\left(3+\frac{2}{9}\pi\right)$ (b) $x^2\left(3+\frac{2}{9}\pi\right)$

(c) $2x^2\left(3+\frac{\pi}{9}\right)$ (d) $3x^2\left(3+\frac{2}{9}\pi\right)$

PART B

Section III

All questions are compulsory. In case of internal choices attempt any one.

19. If $y=|x-x^2|$, then find $\frac{dy}{dx}$ at $x=1$.

20. Find the slope of the normal to the curve $x=1-a\sin\theta,\ y=b\cos^2\theta$ at $\theta=\frac{\pi}{2}$.

Or

Show that for $a\geq 1$, $f(x)=\sqrt{3}\sin x-\cos x-2ax+b$ is decreasing on R.

21. Show that all the positive integral powers of a symmetric matrix are symmetric.

Or If $A=\begin{bmatrix}3 & 0 & -1\\ 2 & 3 & 0\\ 0 & 4 & 1\end{bmatrix}$, then find $|\text{adj}\ (\text{adj}\ A)|$.

22. Show that the three points $A(1,-2,-8)$, $B(5,0,-2)$ and $C(11,3,7)$ are collinear, and also find the ratio in which B divides AC.

23. Solve the following equation.

$$\tan^{-1}\sqrt{x^2+x}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$$

24. Find the probability distribution of number of doublets in three throws of a pair of dice.

25. Find $|\vec{a}\times\vec{b}|$, if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$.

26. Evaluate $\int\frac{1}{\sqrt{9+8x-x^2}}dx$

Or

Evaluate $\int x\log x\,dx$

27. Find the area bounded by the curve $y^2=8x$ and the line $x=4$.

28. Evaluate $\int_1^2\frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}}dx$.

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. If $f:R\rightarrow R$ is a function defined by $f(x)=2x^3-5$, then show that the function f is a bijective function.

30. Evaluate $\int\frac{1}{3x^2+5x+7}dx$.

Or Evaluate $\int\frac{xe^x}{(x+1)^2}dx$.

31. Show that $\int_0^{\pi/2}f(\sin 2x)\sin x\,dx=\sqrt{2}\int_0^{\pi/4}f(\cos 2x)\cos x\,dx$.

32. Find the area of the region bounded by $y=-1$, $y=2$, $x=y^3$ and $x=0$.

33. Solve the following differential equation.

$$\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$$

34. If $y=e^{a\cos^{-1}x}, -1\le x\le 1$, then show that

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0.$$

Or If $y=e^x\sin x$, then prove that

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$$

35. Find the value of a, if the function $f(x)$ defined by $f(x)=\begin{cases}2x-1, & x<2\\ a, & x=2\\ x+1, & x>2\end{cases}$

Section V

All questions are compulsory. In case of internal choices attempt any one.

36. Solve the given LPP maximise $Z=24x+18y$. Subject to constraints $2x+3y\le 10, x\ge 0, y\ge 0, 3x+2y\le 10$.

Or Solve the given LPP

Maximise $(z)=100x+120y$

Subject to constraints,

$$2x+3y\le 30$$

$$3x+y\le 17$$

$$x, y\ge 0$$

37. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

Or

If $A=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$, prove that

$$A^n=\begin{bmatrix}3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\end{bmatrix}, n\in N.$$

38. A variable plane which remains at a constant distance $3p$ from the origin cut the coordinate axes at A, B and C. Show that the locus of the centroid of ΔABC is $x^{-2}+y^{-2}+z^{-2}=p^{-2}$.

Or

If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, find the value of k and hence, find the equation of the plane containing these lines.