Marking Scheme (Mathematics XII 2017-18)

Sr. No.	Answer	Mark(s)
Section A		
1.	$\left[(1,3)\right]=\{(x,y)\in A\times A: x+3=y+1\}=\{(x,y)\in A\times A: y-x=2\}=\{(1,3),(2,4)\}$	[1]
2.	-15	[1]
3.	$\vec{a}=\hat{i}$, $\vec{b}=\hat{j}$. (or any other correct answer)	[1]
4.	$(1*2)*3=2^2*3=2^{12}, 1*(2*3)=1*2^6=2^{64}\ \therefore (1*2)*3\neq 1*(2*3).$ Hence, $*$ is not associative.	[1]
Section B		
5.	$\Rightarrow 4\sin^{-1}x+\frac{\pi}{2}-\sin^{-1}x=\pi$ $\Rightarrow 3\sin^{-1}x=\frac{\pi}{2}\Rightarrow \sin^{-1}x=\frac{\pi}{6}\Rightarrow x=\sin\frac{\pi}{6}=\frac{1}{2}$	[1] [1]
6.	$\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}^{-1}=\frac{1}{9-10}\begin{bmatrix}-3 & -2\\ -5 & -3\end{bmatrix}=\begin{bmatrix}3 & 2\\ 5 & 3\end{bmatrix}$ $P=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}\begin{bmatrix}3 & 2\\ 5 & 3\end{bmatrix}=\begin{bmatrix}13 & 8\\ 1 & 1\end{bmatrix}$	[1 + ½] [½]
7.	Let $\cos^{-1}x=\theta$. Then $\forall x\in\left[\frac{1}{2},1\right], \theta\in\left[0,\frac{\pi}{3}\right], x=\cos\theta$ The given expression on LHS $=\theta+\cos^{-1}\left[\frac{\cos\theta}{2}+\frac{\sqrt{3}\sin\theta}{2}\right]=\theta+\cos^{-1}\left[\cos(\theta-\frac{\pi}{3})\right]=\theta+\cos^{-1}\left[\cos(\frac{\pi}{3}-\theta)\right]$ $=\theta+\frac{\pi}{3}-\theta \quad \left(\because 0\le\frac{\pi}{3}-\theta\le\frac{\pi}{3}\right)$ $=\frac{\pi}{3}=RHS$	[½] [1] [½]
8.	Let $y=\frac{1}{x^2}$. Then $\frac{dy}{dx}=\frac{-2}{x^3}$. $dy=(\frac{dy}{dx})_{x=2}\times\Delta x=\frac{-2}{2^3}\times 0.002=-0.0005.$ y decreases by 0.0005.	[1/2] [1] [1/2]
9.	$\int e^x\frac{\sqrt{1+\sin 2x}}{1+\cos 2x}dx=\int e^x\frac{\sqrt{(\sin x+\cos x)^2}}{2\cos^2 x}dx$ $=\frac{1}{2}\int e^x(\frac{\sin x}{\cos^2 x}+\frac{\cos x}{\cos^2 x})dx=\frac{1}{2}\int e^x(\sec x+\sec x\tan x)dx$ $=\frac{1}{2}e^x\sec x+c\ \left[\because \int e^x(f(x)+f'(x))dx=e^x f(x)+c\right]$	[1/2] [1/2] [1]

10.	$ax^2+by^2=1 \Rightarrow 2ax+2byy_1=0 \Rightarrow ax+byy_1=0$ (1) $\Rightarrow a+b[yy_2+{y_1}^2]=0 \Rightarrow a=-b[yy_2+{y_1}^2]$ (2) Substituting this value, for a in the equation (1), we get, $-b[yy_2+{y_1}^2]x+byy_1=0 \Rightarrow x[yy_2+{y_1}^2]=yy_1$. Hence verified	[1/2] [1] [1/2]
11.	$\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}-2\hat{j}+\hat{k}, \vec{a}\cdot\vec{b}=5, \lvert\vec{b}\rvert=\sqrt{6}.$ The required Projection (vector) of $\vec{a}$ on $\vec{b}=\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert^2}\vec{b}$ $=\frac{5}{6}(\hat{i}-2\hat{j}+\hat{k}).$	[1/2] [1] [1/2]
12.	$P(B/A)=\frac{P(A\cap B)}{P(A)} \Rightarrow P(A\cap B)=0.4\times 0.6=0.24$ $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{0.24}{0.80}=\frac{3}{10}$	[1] [1]
Section C		
13.	$Let\ \Delta_1=\begin{vmatrix} a^3-1 & 0 & a-a^4 \\ 0 & a-a^4 & a^3-1 \\ a-a^4 & a^3-1 & 0 \end{vmatrix}=\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$ Where C_{ij} = the cofactor of a_{ij} and a_{ij} = the (i, j)th element of determinant Δ. $We\ know\ that\ \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}=\Delta^2$ $\therefore\ \Delta_1=\Delta^2=(-4)^2=16$	[2] [1] [1]
14.	Since, f is differentiable at 1, f is continuous at 1. Hence, $\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}(2x+1)=3$ $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}(ax^2+b)=a+b$ f(1) = 3 As f is continuous at 1, we have a + b =3 .... (1) $Lf'(1)=\lim_{h\to 0^+}\frac{f(1-h)-f(1)}{-h}=\lim_{h\to 0^+}\frac{a(1-h)^2+b-3}{-h}$ $=\lim_{h\to 0^+}\frac{a+ah^2-2ah+b-3}{-h}=\lim_{h\to 0^+}(-ah+2a)$ (using (1)) = 2a $Rf'(1)=\lim_{h\to 0^+}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0^+}\frac{2(1+h)+1-3}{h}=2.$ As f is differentiable at 1, we have 2 a = 2, i. e., a = 1 and b = 2. **OR**	[1] [1/2] [1/2] [1/2] [1/2] [1/2] [1/2]

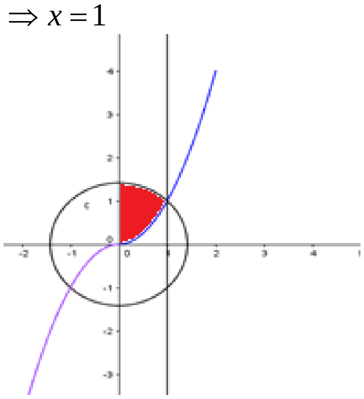
	$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x+\sin x}{\sin(a+1)x}$	[1/2]
	$= \lim_{x\to 0^-} \frac{1+\frac{\sin x}{x}}{\frac{\sin(a+1)x}{(a+1)x}(a+1)} = \frac{2}{a+1}$	[1/2]
	$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 2\frac{e^{\sin bx}-1}{bx}$	[1/2]
	$= \lim_{x\to 0^+} 2\frac{e^{\sin bx}-1}{\sin bx}\times\frac{\sin bx}{bx} = 2$ f(0) = 2.	[1/2]
	For the function to be continuous at 0, we must have $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0)$	[1/2] [1/2]
	i.e., we must have $\frac{2}{a+1} = 2 \Rightarrow a = 0;$ b may be any real number other than 0.	[1/2] [1]
15.	$y = \log(\sqrt{x}+\frac{1}{\sqrt{x}})^2 = 2\log(\frac{x+1}{\sqrt{x}}) = 2[\log(x+1)-\frac{1}{2}\log x]$	[1]
	$y_1 = 2[\frac{1}{x+1}-\frac{1}{2}\times\frac{1}{x}] = \frac{x-1}{x(x+1)}$ (1)	[1]
	$y_2 = \frac{x(x+1)-(x-1)(2x+1)}{x^2(x+1)^2} = \frac{-x^2+2x+1}{x^2(x+1)^2}$	[1]
	$\Rightarrow x(x+1)^2 y_2 = \frac{-x^2+2x+1}{x} = \frac{2x-(x+1)(x-1)}{x} = 2-(x+1)^2 y_1$ (using (1)) $\Rightarrow x(x+1)^2 y_2 + (x+1)^2 y_1 = 2.$ Hence, proved.	[1]
16.	When y = 0, we have (x – 1) (x^2 + x + 1) (x – 2) = 0, i.e., x = 1 or 2.	[1/2]
	$\frac{dy}{dx} = x^3-1+(x-2)3x^2 = 4x^3-6x^2-1$	[1/2]
	$(\frac{dy}{dx})_{(1,0)} = -3$	[1/2]
	$(\frac{dy}{dx})_{(2,0)} = 7$.	[1/2]
	The required equations of the tangents are y – 0 = -3 (x – 1) or, y = -3x + 3 and y – 0 = 7(x – 2) or, y = 7x – 14.	[2]
	OR	
	Domain f = $(-1,\infty)$ $f'(x) = \frac{-3}{1+x}+\frac{4}{(2+x)}+\frac{4}{(2+x)^2} = \frac{x(x+4)}{(1+x)(2+x)^2}$.	[1]
	$f'(x) = 0 \Rightarrow x = 0$ $[x \neq -4$ as $-4 \notin (-1,\infty)]$.	[1]
	In (-1, 0), $f'(x) = \frac{(-ve)(+ve)}{(+ve)(+ve)} = -ve$. Therefore, f is strictly decreasing in (-1, 0].	[1]
	In $(0,\infty)$, $f'(x) = +ve$. Therefore, f is strictly increasing in $[0,\infty)$.	[1]

17.	We have $C(x) = x^3 - 45x^2 + 600x, 10 \le x \le 20$. For the time being we may assume that the function C(x) is continuous in [10, 20]. $C'(x) = 3x^2 - 90x + 600 = 3(x-10)(x-20)$ $C'(x) = 0 \ if \ \ x = 10 \ or \ \ x = 20.$ But, 10, 20 $\notin (10, 20)$. Therefore, the maximum or the minimum value will occur at the points. $C(10) = 2500, C(20) = 2000.$ Hence, the person must place the order for 20 trees and the least amount to be spent = Rs 2000. **Value: The person cares for a healthy environment despite being economically constrained.**	[1] [1] [1] [1]
18.	$\int \frac{\sec x}{1+\cos ecx} dx = \int \frac{\sin x}{\cos x(1+\sin x)} dx = \int \frac{\sin x \cos x}{(1+\sin x)^2 (1-\sin x)} dx$ $= \int \frac{t}{(1+t)^2(1-t)} dt \ [\sin x = t \Rightarrow \cos x dx = dt]$ $\frac{t}{(1+t)^2(1-t)} = \frac{A}{1+t} + \frac{B}{(1+t)^2} + \frac{C}{1-t} \Rightarrow t = A(1+t)(1-t) + B(1-t) + C(1+t)^2$ (an identity) Put t = -1, - 1 = 2 B, i.e., B = -½. Put, t = 1, 1 = 4C, i.e., C = ¼. Put t = 0, 0 = A + B + C, which gives A = $\frac{1}{4}$. Therefore the required integral $= \frac{1}{4}\int \frac{1}{1+t} dt + \frac{-1}{2}\int \frac{1}{(1+t)^2} dt + \frac{1}{4}\int \frac{1}{(1-t)} dt$ $= \frac{1}{4}\log\|1+t\| + \frac{-1}{2} \times \frac{-1}{1+t} - \frac{1}{4}\log\|1-t\| + c$ $= \frac{1}{4}\log\|1+\sin x\| + \frac{1}{2} \times \frac{1}{1+\sin x} - \frac{1}{4}\log\|1-\sin x\| + c$	[1] [1+ 1/2] [1 + 1/2]
19.	The given differential equation is $ye^y dx = (y^3 + 2xe^y)dy, y(0) = 1$ or, $\frac{ye^y}{(y^3 + 2xe^y)} = \frac{dy}{dx} \ or, \frac{dx}{dy} + (-\frac{2}{y})x = \frac{y^2}{e^y}$, which is linear in x. I. F. = $e^{\int \frac{-2}{y} dy} = e^{-2\log y} = \frac{1}{y^2}$ Multiplying both sides by the I. F. and integrating, we get, $x\frac{1}{y^2} = \int e^{-y} dy$ $\Rightarrow x\frac{1}{y^2} = -e^{-y} + c \Rightarrow x = -y^2 e^{-y} + cy^2$ (the general solution). When x = 0, y = 1. $0 = -e^{-1} + c \Rightarrow c = \frac{1}{e}.$ Hence, the required particular solution is $x = -y^2 e^{-y} + \frac{y^2}{e}.$ **OR**	[1] [1] [1/2] [1] [1/2]

	The given differential equation is $(x-y)dy=(x+2y)dx \; or, \frac{dy}{dx}=\frac{x+2y}{x-y}=\frac{1+2\frac{y}{x}}{1-\frac{y}{x}}=f(\frac{y}{x}),$ hence, homogeneous. Put y = v x $\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$. The equation becomes $v+x\frac{dv}{dx}=\frac{1+2v}{1-v} \; or, \frac{1-v}{v^2+v+1}dv=\frac{dx}{x}$ $or, \frac{-1}{2}\times\frac{2v+1-3}{v^2+v+1}dv=\frac{dx}{x} \; or, [\frac{2v+1}{v^2+v+1}+\frac{-3}{v^2+2v\times\frac{1}{2}+(\frac{1}{2})^2+\frac{3}{4}}]dv=\frac{-2dx}{x}$ Integrating, we get $\int\frac{2v+1}{v^2+v+1}dv+\int\frac{-3}{(v+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}dv=\int\frac{-2dx}{x}$ or, $\log(v^2+v+1)-\frac{3\times 2}{\sqrt{3}}\tan^{-1}\frac{v+\frac{1}{2}}{\frac{\sqrt{3}}{2}}=-2\log\lvert x\rvert+c$ or, $\log(y^2+xy+x^2)-2\sqrt{3}\tan^{-1}\frac{2y+x}{\sqrt{3}x}=c$ (the general solution).	[1] [1] [2]
20.	$\vec{a}\times(\vec{a}+\vec{b}+\vec{c})=\vec{a}\times\vec{0}$ $\Rightarrow\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{0}\Rightarrow\vec{a}\times\vec{b}=\vec{c}\times\vec{a}$ $\vec{b}\times(\vec{a}+\vec{b}+\vec{c})=\vec{b}\times\vec{0}$ $\Rightarrow\vec{b}\times\vec{c}=\vec{a}\times\vec{b} \therefore \vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ $\left[\vec{a} \;\; \vec{b} \;\; \vec{c}\right]=\vec{a}\cdot(\vec{b}\times\vec{c})=\vec{a}\cdot(\vec{a}\times\vec{b})$ $=0$ [As the scalar triple product of three vectors is zero if any two of them are equal.]	[1] [1] [1/2] [1/2] [1/2] [1/2]
21.	General point on the first line is $(\lambda-2, 2\lambda+3, 4\lambda-1)$. General point on the second line is $(2\mu+1, 3\mu+2, 4\mu+3)$. Direction ratios of the required line are $\langle\lambda-3, 2\lambda+2, 4\lambda-2\rangle$. Direction ratios of the same line may be $\langle 2\mu, 3\mu+1, 4\mu+2\rangle$. Therefore, $\frac{\lambda-3}{2\mu}=\frac{2\lambda+2}{3\mu+1}=\frac{4\lambda-2}{4\mu+2}$ (1) $\Rightarrow\frac{\lambda-3}{2\mu}=\frac{2\lambda+2}{3\mu+1}=\frac{2\lambda-1}{2\mu+1}=k \; (say)$ $\Rightarrow\lambda-3=2\mu k, 2\lambda+2=(3\mu+1)k, \; 2\lambda-1=(2\mu+1)k$ $\Rightarrow\frac{\lambda-3}{2}=\mu k, 2\lambda+2=3\times\frac{\lambda-3}{2}+k, 2\lambda-1=\lambda-3+k$ $\Rightarrow k=\frac{4\lambda+4-3\lambda+9}{2}=\lambda+2\Rightarrow\lambda=9, \mu=\frac{3}{11}$, which satisfy (1). Therefore, the direction ratios of the required line are $\langle 6,20,34\rangle \; or, \langle 3,10,17\rangle$. Hence, the required equation of the line is $\frac{x-1}{3}=\frac{y-1}{10}=\frac{z-1}{17}$.	[1/2] [1/2] [1/2] [1/2] [1] [1/2] [1/2]

22.	Let E_1 = Bag I is chosen, E_2 = Bag II is chosen, E_3 = Bag III is chosen, A = The two balls drawn from the chosen bag are white and red. $P(E_1)=\frac{1}{3}=P(E_2)=P(E_3)$, $P(A\|E_1)=\frac{1}{6}\times\frac{3}{6}\times 2, P(A\|E_2)=\frac{2}{4}\times\frac{1}{4}\times 2, P(A\|E_3)=\frac{4}{9}\times\frac{2}{9}\times 2$. By Bayes's Theorem, the required probability = $P(E_3\mid A)=\frac{P(E_3)\times P(A\|E_3)}{\sum_{i=1}^{3}P(E_i)\times P(A\|E_i)}=\frac{\frac{1}{3}\times\frac{4}{9}\times\frac{2}{9}\times 2}{\frac{1}{3}\times\frac{1}{6}\times\frac{3}{6}\times 2+\frac{1}{3}\times\frac{2}{4}\times\frac{1}{4}\times 2+\frac{1}{3}\times\frac{4}{9}\times\frac{2}{9}\times 2}=\frac{64}{199}$.	[1] [2] [1]
23.	Let X denotes the random variable. Then X = 0, 1, 2. $P(X=0)=\frac{{}^{16}c_2}{{}^{20}c_2}=\frac{60}{95}, P(X=1)=\frac{{}^{4}c_1\times{}^{16}c_1}{{}^{20}c_2}=\frac{32}{95}, P(X=2)=\frac{{}^{4}c_2}{{}^{20}c_2}=\frac{3}{95}$.	[1 + 1/2]
	<table><tr><td>x_i</td><td>p_i</td><td>x_ip_i</td><td>$x_i^2p_i$</td></tr><tr><td>0</td><td>60/95</td><td>0</td><td>0</td></tr><tr><td>1</td><td>32/95</td><td>32/95</td><td>32/95</td></tr><tr><td>2</td><td>3/95</td><td>6/95</td><td>12/95</td></tr><tr><td>total</td><td></td><td>38/95</td><td>44/95</td></tr></table>	[1/2]
	Mean = $\sum_{i=1}^{3}x_ip_i=\frac{38}{95}=\frac{2}{5}$	[1/2]
	Variance = $\sum_{i=1}^{3}x_i^2p_i-(\sum_{i=1}^{3}x_ip_i)^2=\frac{44}{95}-\frac{4}{25}=\frac{144}{475}$.	[1 +1/2]
	Section D	
24.	$f\circ g:\mathbb{R}\to\mathbb{R}$ defined by $f\circ g(x)=f(g(x))=f(x^3+5)=2(x^3+5)-3=2x^3+7$ Let $x_1,x_2\in\mathbb{R}(D_{f\circ g})$ such that $f\circ g(x_1)=f\circ g(x_2)\Rightarrow 2x_1^3+7=2x_2^3+7\Rightarrow x_1^3=x_2^3\Rightarrow x_1=x_2$. Hence, $f\circ g$ is one-one. Let $y\in\mathbb{R}(Codomain_{f\circ g})$. Then for any x $f\circ g$ (x) = y if 2 x^3 + 7 = y, i.e., if, 2 x^3 = y – 7, i.e., x = $\sqrt[3]{\frac{y-7}{2}}$, which $\in\mathbb{R}(D_{f\circ g})$. Hence, for every $y\in\mathbb{R}(Codomain_{f\circ g}),\exists\sqrt[3]{\frac{y-7}{2}}\in\mathbb{R}(D_{f\circ g})$ such that $f\circ g(\sqrt[3]{\frac{y-7}{2}})=y$. Hence, $f\circ g$ is onto. Since, $f\circ g$ is both one-one and onto, it is invertible.	[1] [1] [2] [1/2]

	$(f\circ g)^{-1}:\mathbb{R}\to\mathbb{R}$ defined by $(f\circ g)^{-1}(x)=\sqrt[3]{\frac{x-7}{2}}$	[1]
	$(f\circ g)^{-1}(9)=\sqrt[3]{\frac{9-7}{2}}=1.$	[1/2]
	OR	
	Let $a,b\in\mathbb{R}$ such that a = 0, b $\neq$ 0. Then $a*b=\lvert a\rvert+b=\lvert 0\rvert+b=b, b*a=b, \therefore a*b=b*a$	[1]
	Let $a,b\in\mathbb{R}$ such that a $\neq$ 0, b = 0. Then $a*b=a, b*a=\lvert b\rvert+a=\lvert 0\rvert+a=a, \therefore a*b=b*a$	[1]
	Let $a,b\in\mathbb{R}$ such that a = 0, b = 0. Then $a*b=a=0, b*a=b=0, \therefore a*b=b*a$.	[1]
	Now we need to check whether * is commutative. One more case is needed to be examined. Let $a,b\in\mathbb{R}$ such that a $\neq$ 0, b $\neq$ 0. Then $a*b=\lvert a\rvert+b, b*a=\lvert b\rvert+a$ and $a*b$ may not be equal to $b*a$, e.g., (-1)* 2=3, 2* (-1) = 1, hence, (-1)* 2 $\neq$ 2* (-1). Thus * is not commutative.	[1]
	The element $e\in\mathbb{R}$ will be the identity element for * if a* e = e * a = a for all $a\in\mathbb{R}$. $a*e=a$ provided e = 0 and $e*a=a$ provided e = 0 (As $0*0=0$ and $0*a=\lvert 0\rvert+a=a$ for a $\neq$ 0). Hence, 0 is the identity element for * .	[2]
25.	$\lvert A\rvert=3(3-6)+(-2)(-12-14)+1(12+7)=62\neq 0$.	[1]
	Hence, A^{-1} exists. Let c_{ij} represent the cofactor of (i, j)th element of A. Then, $c_{11}=-3, c_{12}=26, c_{13}=19, c_{21}=9, c_{22}=-16, c_{23}=5, c_{31}=5, c_{32}=-2, c_{33}=-11.$ adjA = $\begin{bmatrix}-3 & 9 & 5\\ 26 & -16 & -2\\ 19 & 5 & -11\end{bmatrix}$ $A^{-1}=\frac{1}{62}\begin{bmatrix}-3 & 9 & 5\\ 26 & -16 & -2\\ 19 & 5 & -11\end{bmatrix}$	[2]
	The given system of equations is equivalent to the matrix equation $A'X=B,\ where\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}14\\ 4\\ 0\end{bmatrix}.$ $\Rightarrow X=(A')^{-1}B=(A^{-1})'B$	[1]
	$=\frac{1}{62}\begin{bmatrix}-3 & 26 & 19\\ 9 & -16 & 5\\ 5 & -2 & -11\end{bmatrix}\begin{bmatrix}14\\ 4\\ 0\end{bmatrix}=\frac{1}{62}\begin{bmatrix}62\\ 62\\ 62\end{bmatrix}=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$ Hence, x = 1, y = 1, z = 1	[2]
	OR	

	$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \ (R_1 \leftrightarrow R_2)$	[1]
	$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \ (R_2 \to R_2 - 2R_1)$	[1]
	$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -2 & 4 & 1 \end{bmatrix} A \ (R_3 \to R_3 - 2R_2)$	[1]
	$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 & -1 \\ -1 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix} A \ (R_2 \to R_2 + R_3, R_1 \to R_1 - R_3)$	[1]
	$\Rightarrow A^{-1} = \begin{bmatrix} 2 & -3 & -1 \\ -1 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix}$	[1]
	$X = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 & -1 \\ -1 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$	[1]
26.	$y = x\lvert x \rvert = \begin{cases} x^2, x \geq 0 \\ -x^2, x < 0 \end{cases}$.	[1]
	Solving $y = x^2, x^2 + y^2 = 2$ simultaneously, $y + y^2 - 2 = 0 \Rightarrow (y+2)(y-1) = 0 \Rightarrow y = 1$ ($y = x^2$ lies in quadrant I). $\Rightarrow x = 1$	[1/2]
		[1]
	The required area = the shaded area = $\int_0^1 (\sqrt{2-x^2} - x^2)dx$	[2]
	$= \frac{1}{2}[x\sqrt{2-x^2} + 2\sin^{-1}\frac{x}{\sqrt{2}}]_0^1 - \frac{1}{3}[x^3]_0^1 = (\frac{1}{6} + \frac{\pi}{4})$ sq units.	[1+1/2]

27.	The given definite integral = I = $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x}{2-\cos 2x}dx + \frac{\pi}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2-\cos 2x}dx$ $f(x) = \frac{x}{2-\cos 2x}, f(-x) = \frac{-x}{2-\cos 2x} = -f(x).$ Hence, f is odd. Therefore, $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x}{2-\cos 2x}dx = 0$	[1]
	$g(x) = \frac{1}{2-\cos 2x}, g(-x) = \frac{1}{2-\cos 2x} = g(x).$ Hence, g is even. Thus $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2-\cos 2x}dx = 2\int_{0}^{\frac{\pi}{4}} \frac{1}{2-\cos 2x}dx.$	[1]
	Hence, I = $\frac{\pi}{2}\int_{0}^{\frac{\pi}{4}} \frac{1}{2-\cos 2x}dx = \frac{\pi}{2}\int_{0}^{\frac{\pi}{4}} \frac{1}{1+2\sin^2 x}dx$ $= \frac{\pi}{2}\int_{0}^{\frac{\pi}{4}} \frac{\sec^2 x}{1+\tan^2 x + 2\tan^2 x}dx = \frac{\pi}{2}\int_{0}^{\frac{\pi}{4}} \frac{\sec^2 x}{1+3\tan^2 x}dx = \frac{\pi}{2}\int_{0}^{1} \frac{1}{1+3t^2}dt$ $[\tan x = t \Rightarrow \sec^2 x dx = dt]$	[2]
	$= \frac{\pi}{2}\times\frac{1}{3}\int_{0}^{1} \frac{1}{(\frac{1}{\sqrt{3}})^2 + t^2}dt = \frac{\pi}{6}\sqrt{3}[\tan^{-1}\sqrt{3}t]_0^1$ $= \frac{\pi}{6}\sqrt{3}[\frac{\pi}{3}] = \frac{\sqrt{3}\pi^2}{18}$	[2]
	OR Let f(x) = $3x^2 - 2x + 4$. Then the given definite integral = $\int_{-2}^{2} f(x)dx = \lim_{\substack{n\to\infty \\ h\to 0}} h\sum_{r=1}^{n} f(-2+rh),\ where\ nh = 4.$	[1]
	$f(-2+rh) = 3(-2+rh)^2 - 2(-2+rh) + 4 = 3r^2h^2 - 14rh + 20$	[1]
	$\sum_{r=1}^{n} f(-2+rh) = 3h^2\sum_{r=1}^{n} r^2 - 14h\sum_{r=1}^{n} r + 20n = 3h^2\times\frac{n(n+1)(2n+1)}{6} - 14h\times\frac{n(n+1)}{2} + 20n$	[2 + 1/2]
	$\int_{-2}^{2} f(x)dx = \lim_{\substack{n\to\infty \\ h\to 0}} h[3h^2\times\frac{n(n+1)(2n+1)}{6} - 14h\times\frac{n(n+1)}{2} + 20n]$ $= \lim_{\substack{n\to\infty \\ h\to 0}}[3\times\frac{nh(nh+h)(2nh+h)}{6} - 14\times\frac{nh(nh+h)}{2} + 20nh]$ $= \lim_{h\to 0}[\frac{4(4+h)(8+h)}{2} - 7\times 4(4+h) + 20\times 4] = 32$	[1 + 1/2]

28.

The general point on the given line $\frac{x-1}{1}=\frac{y-2}{3}=\frac{z+1}{-9}$ [1]

is $(\lambda+1, 3\lambda+2, -9\lambda-1)$. [1]

The direction ratios of the line parallel to the plane x – y +2 z -3 =0 intersecting the given line and passing through the point (-2, 3, -4) are $\langle \lambda+3, 3\lambda-1, -9\lambda+3 \rangle$ [1]

and $(\lambda+3)1+(3\lambda-1)(-1)+(-9\lambda+3)2=0 \Rightarrow \lambda=\frac{1}{2}$. [1]

The point of intersection is $(\frac{3}{2}, \frac{7}{2}, \frac{-11}{2})$. [1]

The required distance = $\sqrt{(\frac{3}{2}+2)^2+(\frac{7}{2}-3)^2+(\frac{-11}{2}+4)^2} = \frac{\sqrt{59}}{2}$ unit. [1]

29.

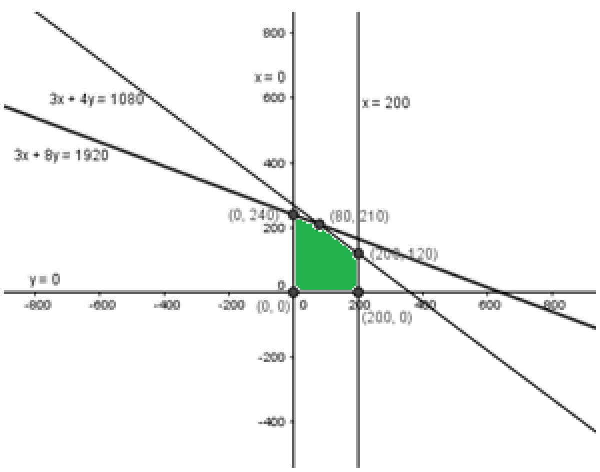
Let x = the number of units of Product 1 to be produced daily

y = the number of units of Product 2 to be produced daily

To maximize P = (9 – 1.2)x + (8 – 0.9)y = 7.8x + 7.1y [1]

subject to the constraints:

$\frac{x}{4}+\frac{y}{3} \le 90, or\ 3x+4y \le 1080, \frac{x}{8}+\frac{y}{3} \le 80, or\ 3x+8y \le 1920, x \le 200, x \ge 0, y \ge 0.$ [2]



[2]

At the point	P
(0, 0)	0
(200, 120)	2412
(0, 240)	1704
(200, 0)	1560
(80, 210)	2115

[1]

The maximum profit = Rs. 2412.